Master's Thesis

Sensitivity Analysis on Prices of Icicled Step Barrier Options

Eunchae Kim

The Graduate School
Sungkyunkwan University
Department of Actuarial Science

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Eunchae Kim

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Approved by

Hangsuck Lee

Major Advisor

This certifies that the Master's Thesis of

Eunchae Kim is approved.

Thesis Committee Chairman

Thesis Committee Member

Thesis Committee Member

The Graduate School
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Abstract

Sensitivity Analysis On Prices of Icicled Step Barrier Options

This paper conducts a sensitivity analysis on a new class of barrier options, termed icicled step barrier options. This class of barrier option, introduced by Lee, Ko, and Song (2018), has embedded barriers whose levels are piecewise constant functions of time and vertical branches attached to the horizontal barriers, referred to as icicles. The icicled barrier options can be utilized in security-linked products with knock-in or knock-out payoff structure. By comparing the prices under different parameter values, calculated by the explicit formulas previously derived in Lee et al (2018), we aim to discover the sensitivity of the icicled barrier option price to each parameter.

Keywords : Barrier Option, Step Barrier Option, Icicled Barrier Option, Sensitivity Analysis

Chapter 1. INTRODUCTION

1. Research Objectives

Barrier option is a type of path-dependent option with its payoff depending on whether the price of its underlying asset reaches a specified level, called a barrier, during the lifetime of the option. Since barrier options always have lower possibilities to pay than standard options, they are no more expensive than standard options. This advantage of lower premium leads to barrier options being widely used in practice, both as a direct investment product or as embedded in equity linked products.

Contrary to typical barrier options, whose barrier level is constant throughout the entire option lifetime, step barrier options have multiple barriers with their levels determined as piecewise constant functions of time. Incorporation of multiple barriers with different levels mitigates inflexibility, the innate limitation of standard options. This led to widespread usage of step barrier options in structured products, such as a step-down ELS, one of the top selling financial products in South Korea.

For insurance companies or financial institutions who aim to sell structured products built with step barrier options, pricing and hedging are inevitable. While several approaches to suggest valuation methods for step barrier options have been made in the literature, less attention has been placed on sensitivities of their prices, which is a starting block for hedging. This paper aims to fill the gap by providing numerical results using closed-form formulas for pricing step barrier options derived in Lee, Ko and Song (2018) and draw conclusions from the sensitivity analysis. Furthermore, this paper also deals with step barrier options with icicled variations as in Lee et al. (2018).

2. Literature Review

First derivations of closed-form pricing formula for barrier options trace back to works by Merton (1973) for a down-and-out call and Rubinstein and Reiner (1991) for all eight types of barrier options. Following these, diversified from the basic form with a single barrier which runs for the entire option lifetime, more complicated barrier options have been introduced to the literature.

Guillaume (2010) introduces analytical formulae for step barrier options, whose barrier level is a piecewise constant function of time. Icicled step barrier option, what we will discuss in details in the following sections, is one variation of the step barrier option.

Lee and Ko (2018) proposed autocallable equity-indexed annuities (EIAs) with icicled step barrier options. On top of proposing the new variation to the financial product with step barrier options, they derived explicit pricing formula for this. This was later developed by Lee, Ko, and Song (2018), by attaching multiple icicles to step barriers, instead of a single icicle to a single barrier. They derived explicit pricing formula in a fully probabilistic method, which can be exploited even by readers without mathematical background in partial differential equation (PDE).

In order for an insurance company or a securities company to deal with products with the new class of barrier options embedded, it should have sufficient information on sensitivity of their payoffs or prices. This paper aims to suggest how this class of option price changes to different levels of market variables or product conditions. Results for sensitivity analysis could work as starting points for designing or hedging the products.

3. Thesis Outline

The remainder of this paper is organized as follows. Chapter 2 first explains step barrier options with icicles. Preliminaries, relevant joint distribution functions and pricing formulas for the previously explained class of barrier option will follow. Chapter 3 introduces one application of this new type of options to equity-linked security, a widely selling investment product in South Korea. Numerical results and sensitivity analysis for prices of icicled step barrier options are presented in chapter 4. Chapter 5 concludes.

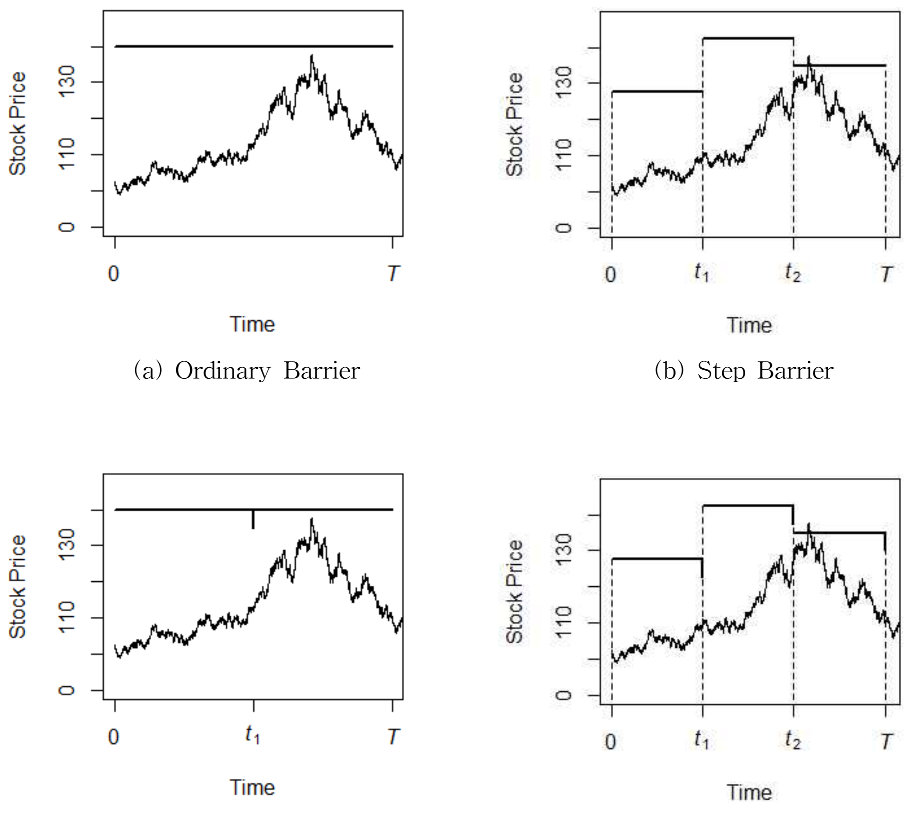
Chapter 2. PRICING STEP BARRIER OPTIONS WITH ICICLED VARIATIONS

1. Step Barrier Options

A barrier option is a path-dependent option, whose payoff depends on whether the underlying asset reaches the predetermined level, during the entire lifetime of the product. An example of ordinary barrier is depicted in (a) of <Figure 2-1>. Whether the path of the underlying asset touches the horizontal barrier, whose level is set at 140 in the example, affects if the option will have effect or not. If the option comes into effect as the underlying reaches the barrier, it is called a knock-in option, while knock-out option refers to an option which expires as the underlying reaches the barrier. If an event of knock-in (knock-out) occurs when the underlying reaches the barrier from below, it is called an up-and-in (up-and-out) option. On the other hand, options which knock-in (knock-out) if the underlying asset reaches the barrier from above are called down-and-in (down-and out) options. One inflexible property of the standard barrier option is that a constant-level barrier runs for the entire lifetime of an option.

A step barrier option is one class of barrier option whose barrier levels are piecewise constant functions of time. (b) of <Figure 2-1> depicts the situation where the maturity consists of three time periods,(0, t_1), (t_1, t_2) and (t_2, T). Unlike standard barrier option, this class of barrier option can provide different levels of

<Figure 2-1> Different Types of Barriers



(a) Ordinary Barrier

(b) Step Barrier

(c) Icicled Barrier

(d) Icicled Step Barrier

barriers for certain periods of time. This is suited to investors who expect the volatility of the underlying would fluctuate as time passes until the maturity. Widespread usage of step barrier options in structured products supports consumers' demands on multiple levels of barriers in barrier options.

Icicled variation of step barrier option, first introduced by Lee and Ko (2018) has both horizontal and vertical barriers as in (d). The vertical barriers, coined as an icicle in the paper gets its name from the resemblance of appearance. If we assume an up-and-out option with the situation in (c) where an icicle is place at time t_1, the underlying asset would exist until maturity only if the underlying asset price is lower than 130 the whole time and if it is lower than 135 at the very moment of t_1. Moreover, we indicate the bottom of a vertical segment as the level of an icicle for up-barrier options and the top of a vertical segment for down-barriers options. To illustrate, the levels of icicles in (d) are 120, 135 and 130 each.

2. Preliminaries

This section provides preliminaries needed for following pricing formulas. Let $S(t)$ denote the time-t value of the underlying asset. For simplicity, assume that this asset pays no dividends. Under the Black-Scholes model,

$$S(t) = S(0)\,e^{X(t)}, \ t \geq 0,$$

where $X(t)$ is a Brownian motion with drift parameter μ and the diffusion parameter σ. Maximum value of the Brownian motion between time s and t will be denoted as

$$M(s,t) = \max\{X(\tau) : s \leq \tau \leq t\}.$$

From the reflection principle, for $0 \leq x \leq m$, the following holds, where $\Phi(\cdot)$ refers to standard normal distribution function. I would refer the readers to Huang and Shiu (2001) for detailed proof.

$$\Pr(X(t) < x, M(0,t) \geq m) = e^{\frac{2\mu}{\sigma^2}m}\Phi\left(\frac{x-2m-\mu t}{\sigma\sqrt{t}}\right).$$

Thus, for $x \leq m$ and $m \geq 0$, the joint distribution of $X(t)$ and $M(0,t)$ is

$$\Pr(X(t) < x, M(0,t) < m) = \Phi\left(\frac{x-\mu t}{\sigma\sqrt{t}}\right) - e^{\frac{2\mu}{\sigma^2}m}\Phi\left(\frac{x-2m-\mu t}{\sigma\sqrt{t}}\right).$$

3. The Joint Distribution Functions

This section shows the jointed probabilities of the events related to $X(t)$ and $M(s,t)$, the logarithmic asset returns and their partial maximums. We use trivariate normal distributions to present events from barrier options with three steps and three icicles. $\Phi_3(\cdot,\cdot,\cdot;\rho_{12},\rho_{13},\rho_{23})$ denotes a trivariate normal distribution function with mean zero and covariance matrix,

$$\Sigma = \begin{bmatrix} 1 & \rho_{12} & \rho_{13} \\ \rho_{12} & 1 & \rho_{23} \\ \rho_{13} & \rho_{23} & 1 \end{bmatrix}.$$

The following formulas state probabilities for situations when the asset price never touches any of the three icicles, but touches the partial horizontal barriers.

(i) For $x_1 \leq m_1$,

$$\Pr(X(t_1) \leq x_1, X(t_2) \leq x_2, X(t_3) \leq x_3, M(0,t_1) > m_1)$$
$$= e^3\Phi_3\left(\frac{x_1-2m_1-\mu t_1}{\sigma\sqrt{t_1}}, \frac{x_1-2m_1-\mu t_1}{\sigma\sqrt{t_1}}, \frac{x_1-2m_1-\mu t_1}{\sigma\sqrt{t_1}}; \sqrt{\frac{t_1}{t_2}}, \sqrt{\frac{t_1}{t_3}}, \sqrt{\frac{t_2}{t_3}}\right)$$

(ii) For $x_1 \le m_2$ and $x_2 \le m_2$,

$$\Pr(X(t_1) \le x_1, X(t_2) \le x_2, X(t_3) \le x_3, M(t_1,t_2) > m_2)$$
$$= e^2 \Phi_3\left(\frac{x_1+\mu t_1}{\sigma\sqrt{t_1}}, \frac{x_2-2m_2-\mu t_2}{\sigma\sqrt{t_2}}, \frac{x_3-2m_2-\mu t_3}{\sigma\sqrt{t_3}}; -\sqrt{\frac{t_1}{t_2}}, -\sqrt{\frac{t_1}{t_3}}, \sqrt{\frac{t_2}{t_3}}\right)$$

(iii) For $x_2 \le m_3$ and $x_3 \le m_3$,

$$\Pr(X(t_1) \le x_1, X(t_2) \le x_2, X(t_3) \le x_3, M(t_2,t_3) > m_3)$$
$$= e^3 \Phi_3\left(\frac{x_1+\mu t_1}{\sigma\sqrt{t_1}}, \frac{x_2+\mu t_2}{\sigma\sqrt{t_2}}, \frac{x_3-2m_2-\mu t_3}{\sigma\sqrt{t_3}}; \sqrt{\frac{t_1}{t_2}}, -\sqrt{\frac{t_1}{t_3}}, -\sqrt{\frac{t_2}{t_3}}\right)$$

(iv) For $x_1 \le \min(m_1,m_2)$ and $x_2 \le m_2$,

$$\Pr(X(t_1) \le x_1, X(t_2) \le x_2, X(t_3) \le x_3, M(0,t_1) > m_1, M(t_1,t_2) > m_2)$$
$$= e^{R(m_2-m_1)} \Phi_3\left(\frac{x_1-2m_1+\mu t_1}{\sigma\sqrt{t_1}}, \frac{x_2-2(m_2-m_1)-\mu t_2}{\sigma\sqrt{t_2}}, \frac{x_3-2(m_2-m_1)-\mu t_3}{\sigma\sqrt{t_3}}; -\sqrt{\frac{t_1}{t_2}}, -\sqrt{\frac{t_1}{t_3}}, \sqrt{\frac{t_2}{t_3}}\right)$$

(v) For $x_1 \le m_1, x_2 \le m_{3,}$ and $x_3 \le m_3$,

$$\Pr(X(t_1) \le x_1, X(t_2) \le x_2, X(t_3) \le x_3, M(0,t_1) > m_1, M(t_2,t_3) > m_3)$$
$$= e^{R(m_3-m_1)} \Phi_3\left(\frac{x_1-2m_1+\mu t_1}{\sigma\sqrt{t_1}}, \frac{x_2-2m_1-\mu t_2}{\sigma\sqrt{t_2}}, \frac{x_3-2(m_3-m_1)-\mu t_3}{\sigma\sqrt{t_3}}; \sqrt{\frac{t_1}{t_2}}, -\sqrt{\frac{t_1}{t_3}}, -\sqrt{\frac{t_2}{t_3}}\right)$$

(vi) For $x_1 \le m_2, x_2 \le \min(m_2,m_3)$, and $x_3 \le m_3$,

$$\Pr(X(t_1) \le x_1, X(t_2) \le x_2, X(t_3) \le x_3, M(t_1,t_2) > m_2, M(t_2,t_3) > m_3)$$
$$= e^{R(m_3-m_1)} \Phi_3\left(\frac{x_1-\mu t_1}{\sigma\sqrt{t_1}}, \frac{x_2-2m_2-\mu t_2}{\sigma\sqrt{t_2}}, \frac{x_3-2(m_3-m_2)-\mu t_3}{\sigma\sqrt{t_3}}; -\sqrt{\frac{t_1}{t_2}}, \sqrt{\frac{t_1}{t_3}}, -\sqrt{\frac{t_2}{t_3}}\right)$$

(vii) For $x_1 \le \min(m_1,m_2), x_2 \le \min(m_2,m_3)$, and $x_3 \le m_3$,

$$
\begin{aligned}
&\Pr(X(t_1) \le x_1, X(t_2) \le x_2, X(t_3) \le x_3, M(0,t_1) > m_2, M(t_1,t_2) > m_2, M(t_2,t_3) > m_3) \\
&= e^{R(m_3 - m_2 - m_1)} \times \\
&\quad \Phi_3\left(\frac{x_1 - 2m_1 - \mu t_1}{\sigma\sqrt{t_1}}, \frac{x_2 - 2(m_2 - m_1) + \mu t_2}{\sigma\sqrt{t_2}}, \frac{x_3 - 2(m_3 - m_2 + m_1) - \mu t_3}{\sigma\sqrt{t_3}}; -\sqrt{\frac{t_1}{t_2}}, \sqrt{\frac{t_1}{t_3}}, -\sqrt{\frac{t_2}{t_3}} \right)
\end{aligned}
$$

Using the formulas given above, for $x_1 \le \min(m_1,m_2), x_2 \le \min(m_2,m_3)$, and $x_3 \le m_3$, we can express the joint distribution of the logarithmic returns at times t_1, t_2, t_3 and their partial maximums as

$$
\begin{aligned}
&\Pr(X(t_1) \le x_1, X(t_2) \le x_2, X(t_3) \le x_3, M(0,t_1) \le m_2, M(t_1,t_2) \le m_2, M(t_2,t_3) \le m_3) \\
&= \Pr(X(t_1) \le x_1, X(t_2) \le x_2, X(t_3) \le x_3) - (i) - (ii) - (iii) + (iv) + (v) + (vi) - (vii) \\
&= \Phi_3\left(\frac{x_1 - \mu t_1}{\sigma\sqrt{t_1}}, \frac{x_2 - \mu t_2}{\sigma\sqrt{t_2}}, \frac{x_3 - \mu t_3}{\sigma\sqrt{t_3}}; \sqrt{\frac{t_1}{t_2}}, \sqrt{\frac{t_1}{t_3}}, \sqrt{\frac{t_2}{t_3}} \right) \\
&\qquad\qquad - (i) - (ii) - (iii) + (iv) + (v) + (vi) - (vii).
\end{aligned}
$$

4. Pricing Formulas for Icicled Step Barrier Options

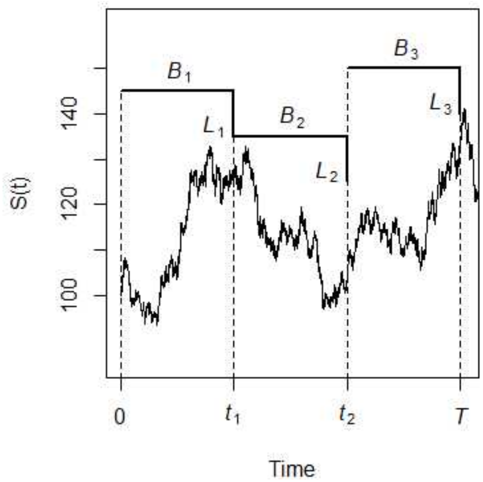
Now we consider eight types of step barrier option with icicles, up-and-in, up-and-out, down-and-in and down-and-out put/call options for pricing. We think of an option with maturity T, with three time points $0 < t_1 < t_2 < t_3 (= T)$. For $i = 1,2,3$, horizontal barrier B_i comes into effect for interval $[t_{i-1}, t_i]$ and icicle L_i, the vertical branch acts at time t_i. This can be visualized as <Figure 2-2>. We denote the risk-free interest rate by r and the strike price by K. To simplify discussion, we define

$$A_u := \{S(t_1) \le L_1, S(t_2) \le L_2, S(t_3) \le L_3, \max\{S(\tau): 0 \le \tau \le t_1\} \le B_1,$$
$$\max\{S(\tau): t_1 \le \tau \le t_2\} \le B_2, \max\{S(\tau): t_2 \le \tau \le t_3\} \le B_3\}$$

$$A_d := \{S(t_1) > L_1, S(t_2) > L_2, S(t_3) > L_3, \max\{S(\tau): 0 \le \tau \le t_1\} > B_1,$$
$$\max\{S(\tau): t_1 \le \tau \le t_2\} > B_2, \max\{S(\tau): t_2 \le \tau \le t_3\} > B_3\}.$$

A_u states an event where the underlying asset never reaches the icicles(L_i) or the horizontal step barriers(B_i) for up-barrier options. This event can be interpreted as no occurrence of any knock-in or knock-out of an option. A_d states

<Figure 2-2>
Icicled step barrier option



the exactly same situation for down-barrier options.

A_u and A_d can be equivalently stated using logarithmic expressions as

$$A_u := \{X(t_1) \le x_1, X(t_2) \le x_2, X(t_3) \le x_3, M(0,t_1) \le m_1, M(t_1,t_2) \le m_2, M(t_2,t_3) \le m_3\}$$

and

$A_d := \{X(t_1) > x_1, X(t_2) > x_2, X(t_3) > x_3, m(0,t_1) > m_1, m(t_1,t_2) > m_2, m(t_2,t_3) > m_3\}$,

where $m(s,t) = Min\{X(\tau): s \le \tau \le t\}$.

<Table 2-1> Payoff Functions of Icicled Step Barrier Options

Option type	Payoff	Option type	Payoff
Up and Out Put (UOP)	$(K-S(T))^+ I(A_u)$	Down and Out Put (DOP)	$(K-S(T))^+ I(A_d)$
Up and In Put (UIP)	$(K-S(T))^+ I(A_u^c)$	Down and In Put (DIP)	$(K-S(T))^+ I(A_d^c)$
Up and Out Call (UOC)	$(S(T)-K)^+ I(A_u)$	Down and Out Call (DOC)	$(S(T)-K)^+ I(A_d)$
Up and In Call (UIC)	$(S(T)-K)^+ I(A_u^c)$	Down and In Call (DIC)	$(S(T)-K)^+ I(A_d^c)$

Using A_u and A_d defined previously, payoff functions for eight types of icicled step barrier options are illustrated in <Table 2-1>. $(S(T)-K)^+$ and $(K-S(T))^+$ are identical to payoffs of ordinary call and put options each. As $I(A_u), I(A_d), I(A_u^c)$, and $I(A_d^c)$ has value 1 only if the underlying asset meets the barrier conditions, the payoff becomes 0 when the conditions are not met. Furthermore, in order to express pricing formulas of the options simpler, let us define functions PA_u and PA_d as following.

$$PA_u(\mu, x_1, x_2, x_3, m_1, m_2, m_3)$$
$$= \Pr(X(t_1) \le x_1, X(t_2) \le x_2, X(t_3) \le x_3, M(0,t_1) \le m_1, M(t_1,t_2) \le m_2, M(t_2,t_3) \le m_3)$$

$$PA_d(\mu, x_1, x_2, x_3, m_1, m_2, m_3)$$
$$= \Pr(X(t_1) > x_1, X(t_2) > x_2, X(t_3) > x_3, M(0,t_1) > m_1, M(t_1,t_2) > m_2, M(t_2,t_3) > m_3)$$

The two functions have a following relationship.

$$PA_d(\mu, x_1, x_2, x_3, m_1, m_2, m_3) = PA_u(-\mu, -x_1, -x_2, -x_3, -m_1, -m_2, -m_3)$$

<Table 2-2> shows pricing formulas for icicled step up-barrier options. Here we let $k = \ln(k/S(0))$ and $a \wedge b = \min\{a,b\}$. In case of up-and-out call, we only consider $K < L_3$, as when the strike price K is larger than the level of the last vertical barrier, L_3, there is no chance the option comes into effect. UOC only pays if the stock price never touches the barrier throughout its lifetime but is larger than the strike price at maturity.

<Table 2-2> Pricing Formulas for Icicled Step Up-Barrier Options

Option type	Price
UOP	$Ke^{-rT}PA_u(r-\frac{\sigma^2}{2},x_1,x_2,x_3\wedge k,m_1,m_2,m_3)-S(0)PA_u(r+\frac{\sigma^2}{2},x_1,x_2,x_3\wedge k,m_1,m_2,m_3)$
UIP	$Ke^{-rT}\left[\Phi(\frac{k-(r-\frac{\sigma^2}{2})T}{\sigma\sqrt{T}})-PA_u(r-\frac{\sigma^2}{2},x_1,x_2,x_3\wedge k,m_1,m_2,m_3)\right]$ $-S(0)\left[\Phi(\frac{k-(r-\frac{\sigma^2}{2})T}{\sigma\sqrt{T}})-PA_u(r+\frac{\sigma^2}{2},x_1,x_2,x_3\wedge k,m_1,m_2,m_3)\right]$
UOC $(K<L_3)$	$S(0)\left[PA_u(r+\frac{\sigma^2}{2},x_1,x_2,x_3,m_1,m_2,m_3)-PA_u(r+\frac{\sigma^2}{2},x_1,x_2,k,m_1,m_2,m_3)\right]$ $-Ke^{-rT}\left[PA_u(r-\frac{\sigma^2}{2},x_1,x_2,x_3,m_1,m_2,m_3)-PA_u(r-\frac{\sigma^2}{2},x_1,x_2,k,m_1,m_2,m_3)\right]$ $S(0)\left[PA_u(r+\frac{\sigma^2}{2},x_1,x_2,x_3,m_1,m_2,m_3)-PA_u(r+\frac{\sigma^2}{2},x_1,x_2,k,m_1,m_2,m_3)\right]$ $-Ke^{-rT}\left[PA_u(r-\frac{\sigma^2}{2},x_1,x_2,x_3,m_1,m_2,m_3)-PA_u(r-\frac{\sigma^2}{2},x_1,x_2,k,m_1,m_2,m_3)\right]$
UIC $(K<L_3)$	$S(0)[\Phi(-\frac{k-(r+\frac{\sigma^2}{2})T}{\sigma\sqrt{T}})-PA_u(r+\frac{\sigma^2}{2},x_1,x_2,x_3,m_1,m_2,m_3)$ $+PA_u(r+\frac{\sigma^2}{2},x_1,x_2,k,m_1,m_2,m_3)]-Ke^{-rT}[\Phi(-\frac{k-(r-\frac{\sigma^2}{2})T}{\sigma\sqrt{T}})$ $-PA_u(r-\frac{\sigma^2}{2},x_1,x_2,x_3,m_1,m_2,m_3)+PA_u(r-\frac{\sigma^2}{2},x_1,x_2,k,m_1,m_2,m_3)]$
UIC $(K>L_3)$	$S(0)\Phi(-\frac{k-(r+\frac{\sigma^2}{2})T}{\sigma\sqrt{T}})-Ke^{-rT}\Phi(-\frac{k-(r-\frac{\sigma^2}{2})T}{\sigma\sqrt{T}})$

Chapter 3. APPLICATIONS OF ICICLED STEP BARRIER OPTIONS

1. Autocallable ELS

Equity-linked security(ELS) is one of the best-selling investment product in Korea. The equity-linked securities refer to financial products whose payoffs track the performance of underlying assets. Vast majorities of ELS selling in Korea has an auto-callable feature, which means the product is automatically called at certain redemption intervals when pre-determined conditions are met, or expire at maturity.

In order to exploit pricing formulas provided in previous chapter, let us assume an ELS product with three time intervals. The first two time points t_1 and t_2 are early redemption dates, which can also be called autocall dates. An investor makes an initial investment F equal to 1 at time 0. At each early redemption date, the underlying index will be checked against the barriers. If the underlying index is higher than the predetermined value at t_1, the initial investment is automatically redeemed with a higher rate of return at the moment. If the first condition is not met, the investor waits until the second autocall date and check whether the early redemption condition is met. Usually this condition at second early redemption date is not higher than the first one. Again, if the condition is met at the moment, the investor receives the initial investment with a high rate of return. If the underlying could not meet both of the conditions, than it will be checked whether the maximum value of the underlying index has ever touched a predetermined barrier. If this last condition is not met, the investor will receive the underlying index as it is. The discounted payoff from this auto-callable product can be illustrated as follows:

$$\text{discounted payoff}=\begin{cases} e^{-rt_1}(1+c_1), & \text{if } X(t_1)>x_1, \\ e^{-rt_2}(1+c_2), & \text{if } X(t_1)\le x_1, X(t_2)>x_2, \\ e^{-rt_3}(1+c_3), & \text{if } X(t_1)\le x_1, X(t_2)\le x_2, m(0,T)>m_1 \\ e^{-rt_3}e^{X(T)}, & otherwise. \end{cases}$$

Price of this auto-callable product can be evaluated as following.

$$\begin{aligned} &e^{-rt_1}(1+c_1)\Pr(X(t_1)>x_1;h^*)+e^{-rt_2}(1+c_2)\Pr(X(t_1)\le x_1, X(t_2)>x_2,;h^*) \\ &+e^{-rt_3}(1+c_3)\Pr(X(t_1)\le x_1, X(t_2)\le x_2, m(0,T)>m;h^*) \\ &+e^{-rt_3}E[e^{X(T)}I(X(t_1)\le x_1, X(t_2)\le x_2, m(0,T)\le m);h^*] \\ &=e^{-rt_1}(1+c_1)\times(I)+e^{-rt_2}(1+c_2)\times(II)+e^{-rt_3}(1+c_3)\times(III)+(IV), \text{ where} \end{aligned}$$

$$(I)=\Pr(X(t_1)>x_1;h^*)=\Phi\left(-\frac{x_1-(r-\sigma^2/2)t_1}{\sigma\sqrt{t_1}}\right),$$

$$\begin{aligned} (II)&=\Pr(X(t_1)\le x_1, X(t_2)>x_2;h^*) \\ &=\Pr(X(t_1)\le x_1)-(X(t_1)\le x_1, X(t_2)\le x_2;h^*) \\ &=\Phi\left(\frac{x_1-(r-\sigma^2/2)t_1}{\sigma\sqrt{t_1}}\right)-\Phi_2\left(\frac{x_1-(r-\sigma^2/2)t_1}{\sigma\sqrt{t_1}},\frac{x_2-(r-\sigma^2/2)t_2}{\sigma\sqrt{t_2}};\sqrt{\frac{t_1}{t_2}}\right). \end{aligned}$$

For (III), we denote $Y(t) = -X(t)$ for $0 \le t \le T$, $y_1 = -x_1, y_2 = -x_2, m_y = -m$, and $M_y(s,t) = \max\{Y(\tau); s \le \tau \le T\}$.

$$
\begin{aligned}
(III) &= \Pr(X(t_1) \le x_1, X(t_2) \le x_2, m(0,T) > m; h^*) \\
&= \Pr(Y(t_1) \ge y_1, Y(t_2) \ge y_2, M_y(0,T) \le m; h^*) \\
&= \Pr(M_y(0,T) \le m; h^*) - \Pr(Y(t_1) < y_1, M_y(0,T) \le m; h^*) \\
&\quad - \Pr(Y(t_2) < y_2, M_y(0,T) \le m; h^*) + \Pr(Y(t_1) < y_1, Y(t_2) < y_2, M_y(0,T) \le m; h^*)
\end{aligned}
$$

$$
\Pr(M_y(0,T) \le m; h^*) = \Phi\left(\frac{m_y + (r - \sigma^2/2)T}{\sigma\sqrt{T}}\right) - e^{-(2r/\sigma^2 - 1)m_y}\Phi\left(\frac{-m_y + (r - \sigma^2/2)T}{\sigma\sqrt{T}}\right),
$$

$$
\begin{aligned}
\Pr(Y(t_i) < y_i, M_y(0,T) \le m; h^*) &= \Phi_2\left(\frac{y_i + (r - \sigma^2/2)t_i}{\sigma\sqrt{t_i}}, \frac{m_y + (r - \sigma^2/2)T}{\sigma\sqrt{T}}; \sqrt{\frac{t_i}{T}}\right) \\
&\quad + \Phi_2\left(\frac{y_i - 2m_y - (r - \sigma^2/2)t_i}{\sigma\sqrt{t_i}}, \frac{m_y + (r - \sigma^2/2)T}{\sigma\sqrt{T}}; -\sqrt{\frac{t_i}{T}}\right) \\
&\quad - e^{-(2r/\sigma^2 - 1)m_y}\Phi_2\left(\frac{y_i - 2m_y + (r - \sigma^2/2)t_i}{\sigma\sqrt{t_i}}, \frac{-m_y + (r - \sigma^2/2)T}{\sigma\sqrt{T}}; \sqrt{\frac{t_i}{T}}\right) \\
&\quad - e^{-(2r/\sigma^2 - 1)m_y}\Phi_2\left(\frac{y_i - (r - \sigma^2/2)t_i}{\sigma\sqrt{t_i}}, \frac{-m_y + (r - \sigma^2/2)T}{\sigma\sqrt{T}}; -\sqrt{\frac{t_i}{T}}\right)
\end{aligned}
$$

for $i = 1,2$.

$$
\begin{aligned}
&\Pr(Y(t_1) < y_1, Y(t_2) < y_2, M_y(0,T) \le m; h^*) \\
&= \Pr(Y(t_1) < y_1, Y(t_2) < y_2, Y(T) < m, M_y(0,t_1) \le m, M_y(0,t_2) \le m, M_y(0,T) \le m; h^*) \\
&= PA_u(-r + \sigma^2/2, -x_1, -x_2, -m, -m, -m, -m).
\end{aligned}
$$

From the factorization formula, (IV) can be restated as

$$
\begin{aligned}
(IV) &= E[e^{X(T)} I(X(t_1) \le x_1, X(t_2) \le x_2, m(0,T) \le m); h^*] \\
&= \Pr(X(t_1) \le x_1, X(t_2) \le x_2, m(0,T) \le m; h^*+1) \\
&= \Pr(X(t_1) \le x_1, X(t_2) \le x_2; h^*+1) - \Pr(X(t_1) \le x_1, X(t_2) \le x_2, m(0,T) > m; h^*+1)
\end{aligned}
$$

whose probabilities can be easily calculated from the preceding formulas, by changing the drift.

2. An ELS with Step Barrier Option Embedded

This section introduces one example of application of icicled step barrier options to ELS. Unlike the auto-callable product assumed in previous section, we will think of a new design that incorporates partial maximums within the subperiods before maturity. Discounted payoff for this product can be illustrated as follows:

$$
\text{discounted payoff} = \begin{cases} e^{-rt_1}(1+c_1), & \text{if } M(0,t_1) > m_1, \\ e^{-rt_2}(1+c_2), & \text{if } M(0,t_1) \le m_1,\ M(t_1,t_2) > m_2, \\ e^{-rT}(1+c_3), & \text{if } M(0,t_1) \le m_1, M(t_1,t_2) \le m_2, M(t_2,T) > m_3 \\ e^{-rT}, & \textit{otherwise} \end{cases}
$$

Example of auto-callable suggested in previous section has early redemption conditions of the underlying index at a moment. However, by using partial maximum conditions, this product scheme can appeal to investors who want their payoff linked to performance of the underlying index over a period, not at one point of time.

The price of this new product can be similarly state as following.

$$e^{-rt_1}(1+c_1)\Pr(M(0,t_1)>m_1;h^*)+e^{-rt_2}(1+c_2)\Pr(M(0,t_1)\le m_1,\ M(t_1,t_2)>m_2,;h^*)$$

$$+e^{-rT}(1+c_3)\Pr(M(0,t_1)\le m_1,\ M(t_1,t_2)\le m_2,\ M(t_2,T)>m_3;h^*)$$

$$+e^{-rT}\Pr(M(0,t_1)\le m_1,\ M(t_1,t_2)\le m_2,\ M(t_2,T)\le m_3;h^*)$$

$$=e^{-rt_1}(1+c_1)\times(I)+e^{-rt_2}(1+c_2)\times(II)+e^{-rT}\big[(1+c_3)\times(III)+(IV)\big].$$

$$(I)=\Pr(M(0,t_1)>m_1;h^*)=\Pr(X(t_1)>m_1;h^*)+\Pr(X(t_1)\le m_1,\ M(0,t_1)>m_1;h^*)$$

$$=\Phi\left(\frac{-m_1+(r-\sigma^2/2)t_1}{\sigma\sqrt{t_1}}\right)+e^{(2r/\sigma^2-1)m_1}\Phi\left(\frac{-m_1-(r-\sigma^2/2)t_1}{\sigma\sqrt{t_1}}\right)$$

$$(II)=\Pr(M(0,t_1)\le m_1,M(t_1,t_2)>m_2;h^*)$$

$$=\Pr(M(0,t_1)\le m_1;h^*)-\Pr(M(0,t_1)\le m_1,M(t_1,t_2)\le m_2;h^*)$$

$$=1-\Pr(M(0,t_1)>m_1;h^*)-\Pr(M(0,t_1)\le m_1,M(t_1,t_2)\le m_2;h^*)$$

$$\begin{aligned}\Pr(M(0,t_1)\le m_1,M(t_1,t_2)\le m_2;h^*)=&\ \Phi_2\left(\frac{m_1-(r-\sigma^2/2)t_1}{\sigma\sqrt{t_1}},\frac{m_2-(r-\sigma^2/2)t_2}{\sigma\sqrt{t_2}};\sqrt{\frac{t_1}{t_2}}\right)\\&-e^{(2r/\sigma^2-1)m_1}\Phi_2\left(\frac{-m_1-(r-\sigma^2/2)t_1}{\sigma\sqrt{t_1}},\frac{m_2-2m_1-(r-\sigma^2/2)t_2}{\sigma\sqrt{t_2}};\sqrt{\frac{t_1}{t_2}}\right)\\&-e^{(2r/\sigma^2-1)m_2}\Phi_2\left(\frac{m_1+(r-\sigma^2/2)t_1}{\sigma\sqrt{t_1}},\frac{-m_2-(r-\sigma^2/2)t_2}{\sigma\sqrt{t_2}};-\sqrt{\frac{t_i}{T}}\right)\\&+e^{(2r/\sigma^2-1)(m_2-m_1)}\Phi_2\left(\frac{-m_1+(r-\sigma^2/2)t_1}{\sigma\sqrt{t_1}},\frac{-m_2+2m_1-(r-\sigma^2/2)t_2}{\sigma\sqrt{t_2}};-\sqrt{\frac{t_i}{T}}\right)\end{aligned}$$

$$(III)=\Pr(M(0,t_1)\le m_1,\ M(t_1,t_2)\le m_2,\ M(t_2,T)>m_3;h^*)$$

$$
\begin{aligned}
&= \Pr(M(0,t_1) \le m_1,\, M(t_1,t_2) \le m_2; h^*) \\
&\qquad - \Pr(M(0,t_1) \le m_1,\, M(t_1,t_2) \le m_2,\, M(t_2,T) \le m_3; h^*) \\
&= \Pr(M(0,t_1) \le m_1,\, M(t_1,t_2) \le m_2; h^*) - (IV)
\end{aligned}
$$

$$
\begin{aligned}
(IV) &= \Pr(M(0,t_1) \le m_1,\, M(t_1,t_2) \le m_2,\, M(t_2,T) \le m_3; h^*) \\
&= PA_u(r - \frac{\sigma^2}{2}, m_1, m_2, m_3, m_1, m_2, m_3)
\end{aligned}
$$

Chapter 4. NUMERICAL RESULTS AND SENSITIVITY ANALYSIS

With explicit pricing formulas provided in the previous chapters, calculation of icicled barrier option prices becomes straightforward for a given set of parameter values. This chapter will provide icicled step barrier options prices calculated with different levels of parameters and discover the impact of each variable and its parameter sensitivities. The following numerical calculations are performed using the R packages.

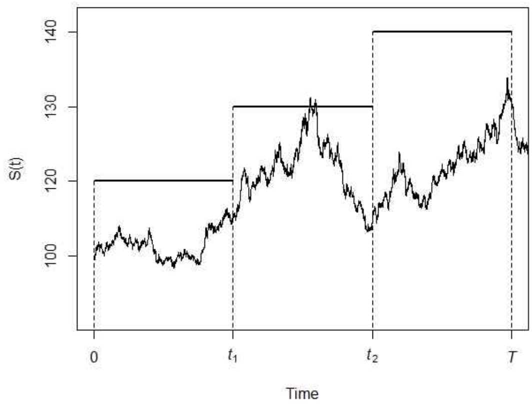
1. Numerical Results for Step Barrier Option Prices

<Table 4-1> The step barrier option prices
with S(0)=100, K=100, r=3%, σ=20%, t_1=0.5, t_2=1, T=t_3=1.5

	B_1	B_2	B_3	L_1	L_2	L_3	(1)UOC	(2)UIC	(3)UOP	(4)UIP
(i)	100	100	100	100	100	100	0	11.8929	0	7.4927
(ii)	∞	∞	∞	∞	∞	∞	11.8929	0	7.4927	0
(iii)	120	120	120	120	120	120	0.7263	11.1666	6.8804	0.6122
(iv)	120	130	140	120	130	140	1.6625	10.2304	7.0676	0.4250
(v)	140	140	140	140	140	140	4.4136	7.4793	7.4735	0.0192
(vi)	120	140	160	120	140	160	1.8311	10.0618	7.0897	0.4030
(vii)	∞	140	∞	∞	140	∞	7.7007	4.1923	7.4740	0.0187

Consistent with previous discussions, numerical studies in this section will continue to assume three terms and set them as t_1=0.5, t_2=1, and T=t_3=1.5. The barrier levels are changeable at each term end: end of a half year (t_1=0.5) or end of a year (t_2=1). The icicles can be set at end of each barrier, t_1=0.5, t_2=1 and T=t_3=1.5.

<Figure 4-1> Barrier condition (iv)
of <Table 3-1>



<Table 4-1> demonstrates prices of options without icicles, assuming S(0)=100, K=100, r=3%, σ=20%, t_1=0.5, t_2=1, T=t_3=1.5. Step barrier options are special cases of icicled step barrier options, when levels of icicles equal to the horizontal barriers. Rows (i) and (ii) show prices of Vanilla call and put options. By setting the level of first barrier equal to the initial stock price S(0), options in row (i) knock-out (for up-and-out) or knock-in (for up-and-in) at the beginning of their lifetime. On the other hand, since options in row (ii) cannot reach the barriers set at infinity, up-and-out options are equivalent to ordinary call or put options, while up-and-in

options do not pay regardless of stock price movement, becoming zero value. Rows (iii) and (v) illustrate ordinary barrier options, with a single barrier during the option duration. Rows (iv) and (vi) show prices of step barrier options, with different level of barriers for each of three terms. <Figure 4-1> illustrates a step barrier option with barrier options from row (iv). Row (vii) shows an example of partial barrier option with monitoring period of [0.5, 1].

A barrier option must be worth less than or equal to one without barrier, due to less probability to pay. The results in the table are consistent with this property. Furthermore, there is a parity relationship for barrier options having the same barrier: Knock-in-option + Knock-out-option = Ordinary option. We can notice values of columns (1) and (2) add up to vanilla call option price (=11.8929) and those of (3) and (4) add up to vanilla put option price (=7.4927), respectively.

Let's look into details. As rows (iii) and (v) are both examples of ordinary barrier options, we can observe price change made by different levels of barriers. As the barrier level rises from 120 to 140, prices of UOC and UOP became more expensive, while those of UIC and UIP became cheaper. The rise in barrier level leads to lower possibilities of stock prices reaching the up-barrier, for both knock-in or knock-out conditions. Therefore, up-and-out options bear less risk to lose value, while up-and-in options bear more risk that the option might not go into effect.

2. Numerical Results for Icicled Step Barrier Option Prices

<Table 4-2> illustrates the impact of different levels of barriers and icicles, interest rate, volatility and strike price on step barrier option prices. Comparing between (i) and (ii), (iii) and (iv) gives insights for impact of horizontal barrier level on option prices. On the other hand, we can note the impact of vertical icicles on option prices by comparing between (ii) and (iii) or (iv) and (v). Basically, mechanisms of how barriers and icicles affect option prices are similar. Either vertical or horizontal, higher level of barriers leads to lower possibilities to knock-in or knock-out, leading to up-and-out options becoming more valuable and up-and-in options becoming less valuable.

The option prices for risk-free rate of 3% and 4% show that as r rises, UOC and UIC prices rise while those of UOP and UIP falls. This can be attributed to properties of ordinary options, where call option prices have negative sensitivities and put option prices have positive sensitivities with respect to r.

Volatility, measured by standard deviation of stock log-return, bring complicated consequences on option prices. From the table, we can note that most UIC, UOP and UIP prices rise as sigma rises, while UOC prices drop. Without icicles or barriers, option prices are known to rise as volatility rises. However, higher volatility of logreturn seems to raise the possibilities of knocking-out for UOC, leading to lower prices. Higher strike price lowers value of call option and increases value of put option. The effect of strike price is identical for icicled step barrier options, as the structure of payoff at maturity is the same. Impact of risk-free rate and volatility on option prices will be discovered in more details, in next section.

<Table 4-2> The icicled step barrier option prices with different levels of barriers and icicles assuming S(0)=100, t_1=0.5, t_2=1, T=t_3=1.5

			K=100									
r	σ		B_1	B_2	B_3	L_1	L_2	L_3	(1)UOC	(2)UIC	(3)UOP	(4)UIP
3%	20%	(i)	120	130	140	110	120	130	2.1868	9.7062	6.8517	0.6410
		(ii)	130	140	150	110	120	130	2.4103	9.4826	6.9962	0.4964
		(iii)	130	140	150	120	130	140	4.6258	7.2671	7.3714	0.1213
		(iv)	140	150	160	120	130	140	4.8730	7.0199	7.3930	0.0996
		(v)	140	150	160	130	140	150	6.9729	4.9200	7.4748	0.1787
	30%	(i)	120	130	140	110	120	130	1.0186	15.5425	9.7192	2.4416
		(ii)	130	140	150	110	120	130	1.3984	15.1627	10.6529	1.5079
		(iii)	130	140	150	120	130	140	2.3879	14.1627	11.2133	0.9475
		(iv)	140	150	160	120	130	140	2.9301	14.1731	11.5633	0.5975
		(v)	140	150	160	130	140	150	4.1503	13.6310	11.8196	0.3412
4%	20%	(i)	120	130	140	110	120	130	2.2258	10.4308	6.2348	0.5982
		(ii)	130	140	150	110	120	130	2.4548	10.2018	6.3686	0.4644
		(iii)	130	140	150	120	130	140	4.7560	7.9006	6.7191	0.1140
		(iv)	140	150	160	120	130	140	5.0140	7.4626	6.7394	0.0937
		(v)	140	150	160	130	140	150	7.2324	5.4242	6.8162	0.0169
	30%	(i)	120	130	140	110	120	130	1.0225	16.2270	9.1037	2.3222
		(ii)	130	140	150	110	120	130	1.4046	15.8449	9.9870	1.4390
		(iii)	130	140	150	120	130	140	2.4079	14.8415	10.5212	0.9048
		(iv)	140	150	160	120	130	140	2.9571	14.2924	10.8540	0.5719
		(v)	140	150	160	130	140	150	4.2027	13.0468	11.0991	0.3268

			K=120									
r	σ		B_1	B_2	B_3	L_1	L_2	L_3	(1)UOC	(2)UIC	(3)UOP	(4)UIP
3%	20%	(i)	120	130	140	110	120	130	0.1408	4.5561	16.5923	2.8243
		(ii)	130	140	150	110	120	130	0.1661	4.5308	17.0691	2.3474
		(iii)	130	140	150	120	130	140	0.7241	3.9727	18.7026	0.7140
		(iv)	140	150	160	120	130	140	0.7973	3.8996	18.8055	0.6111
		(v)	140	150	160	130	140	150	1.6334	3.0635	19.2752	0.1414
	30%	(i)	120	130	140	110	120	130	0.0680	9.2872	17.8386	6.2363
		(ii)	130	140	150	110	120	130	0.1071	9.2480	19.8531	4.2218
		(iii)	130	140	150	120	130	140	0.3868	8.9684	21.3043	2.7706
		(iv)	140	150	160	120	130	140	0.5237	8.8315	22.1792	1.8957
		(v)	140	150	160	130	140	150	1.0201	8.3351	22.9300	1.1448
4%	20%	(i)	120	130	140	110	120	130	0.1461	4.9898	15.4281	2.7196
		(ii)	130	140	150	110	120	130	0.1724	4.9635	15.8814	2.2663
		(iii)	130	140	150	120	130	140	0.7597	4.3762	17.4536	0.6741
		(iv)	140	150	160	120	130	140	0.8368	4.2991	17.5530	0.5946
		(v)	140	150	160	130	140	150	1.7293	3.4066	18.0092	0.1385
	30%	(i)	120	130	140	110	120	130	0.0689	9.7912	16.8597	6.0121
		(ii)	130	140	150	110	120	130	0.1085	9.7516	18.7866	4.0852
		(iii)	130	140	150	120	130	140	0.3935	9.4666	20.1873	2.6845
		(iv)	140	150	160	120	130	140	0.5330	9.3271	21.0302	1.8416
		(v)	140	150	160	130	140	150	1.0422	8.8179	21.7580	1.1138

3. Sensitivity Analysis of Icicled Step Barrier Option Prices

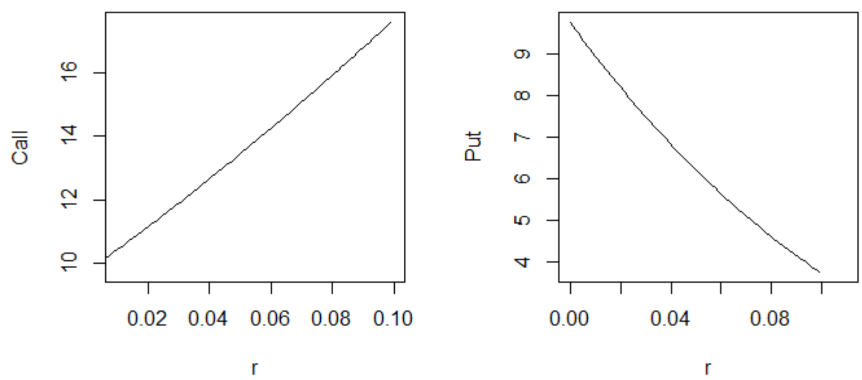
To get some insights into the parameter sensitivities of icicled step barrier option prices, this subsection is devoted to demonstrating numerical results of prices under different levels of parameter values including the risk-free rate r, stock return volatility σ and strike price K.

Throughout this section, we will assume $S(0)=100$, $t_1=0.5$, $t_2=1$, $T=t_3=1.5$, and $B_1=130$, $B_2=140$, $B_3=150$, $L_1=110$, $L_2=120$, $L_3=130$ if not stated otherwise.

1) Variation of Option Price with Interest Rate

The rate of change of an option price with respect to the interest rate, is called rho in Greeks. For ordinary options, call option price increases and put option price

<Figure 4-2> Ordinary option prices
under different levels of risk-free rate, with K=100, σ=20%



decreases as r rises. This relationship can be explicitly shown by differentiating option price calculated by Black-Scholes formula with respect to r, as follows. c and p stand for price of ordinary call option and put option, respectively, and N(·) denotes cumulative distribution function for standard normal distribution. <Figure 4-2> visually depicts this relationship.

$$c = S_0 N(d_1) - Ke^{-rT} N(d_2)$$

$$p = Ke^{-rT} N(-d_2) - S_0 N(-d_1)$$

$$where\ \ d_1 = \frac{\ln(S_0/K) + (r+\sigma^2/2)T}{\sigma\sqrt{T}},$$

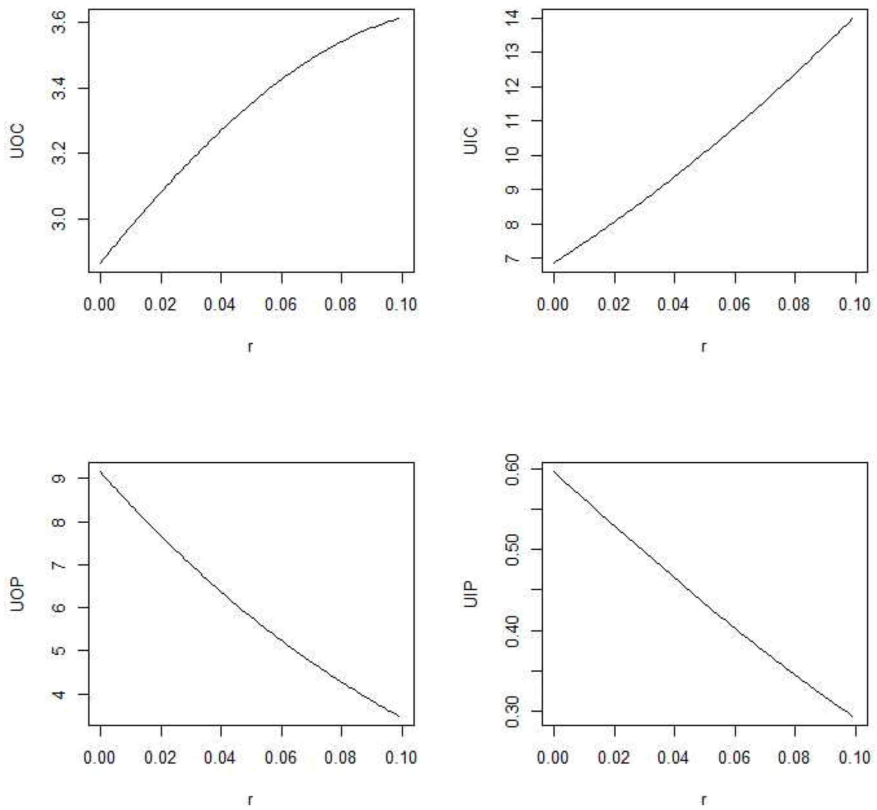
$$d_2 = \frac{\ln(S_0/K) + (r-\sigma^2/2)T}{\sigma\sqrt{T}}$$

$$\frac{\partial c}{dr} = KTe^{-rT} N(d_2) \geq 0$$

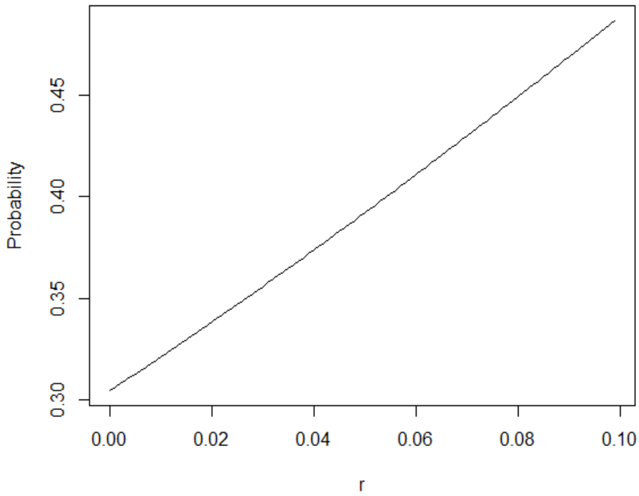
$$\frac{\partial p}{dr} = -KTe^{-rT} N(-d_2) \leq 0$$

The trend of option price shown in <Figure 4-3> corresponds to this traditional understandings in rho. What we can find in details would be the convexity, or the second derivatives of the option price with respect to r. When we compare UIC to UOC, we notice that price function is convex for UIC and concave for UOC. The risk-free rate r mainly affects the discounting factor in pricing function of options. However, as we set drift parameter μ of the Brownian motion equal to $r-\frac{1}{2}\sigma^2$, increase in risk-free rate also raises the drift of the process. In other words, when other conditions stays still, the probability of knocking-in or knocking-out also increases in higher interest rate conditions, as in <Figure 4-4>. Therefore, UIC price increases faster than UOC. In a similar context but in the opposite direction, UOP price decreases faster than UIP, as UOP has to bear higher possibilities to knock-out.

<Figure 4-3> Icicled step barrier option prices (up=barrier) under different levels of risk-free rate, with K=100, σ=20%,



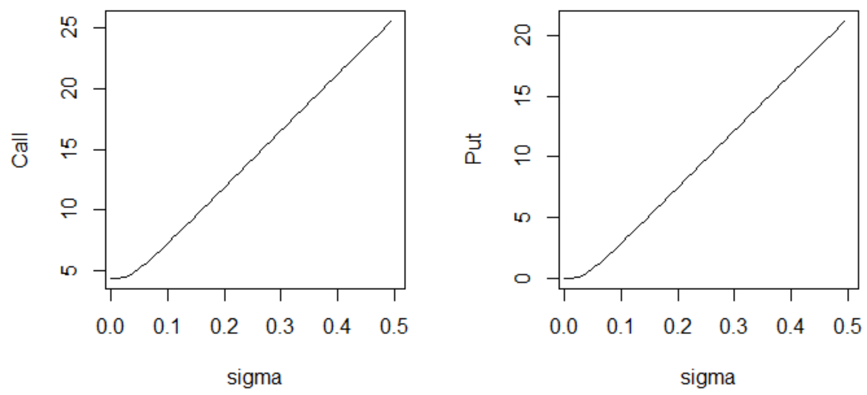
<Figure 4-4> Risk-neutral probability of knocking-in or knocking-out before maturity with K=100, σ=20%,



2) Variation of Option Price with Volatility

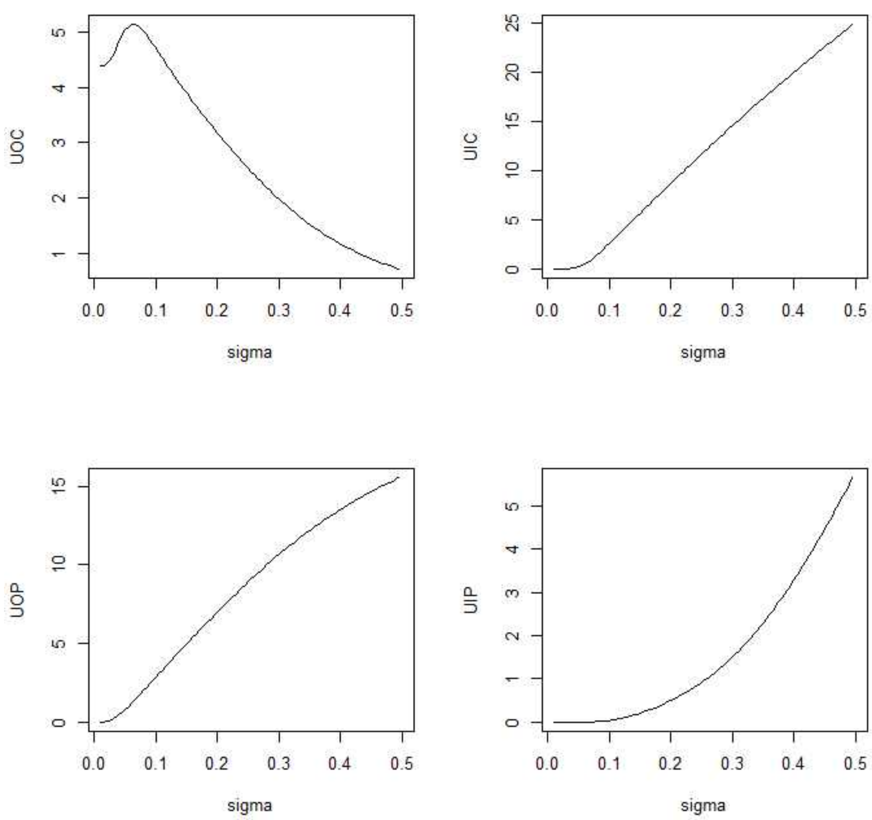
For ordinary options, option price is a non-decreasing function of volatility, as presented in <Figure 4-5>. But we can note that for UOC, the price peaks when sigma is around 0.1 and drops in <Figure 4-6>, which depicts variation of icicled step barrier option price with different level of volatility.

<Figure 4-5> Ordinary option prices
under different levels of volatility, with K=100, r=3%

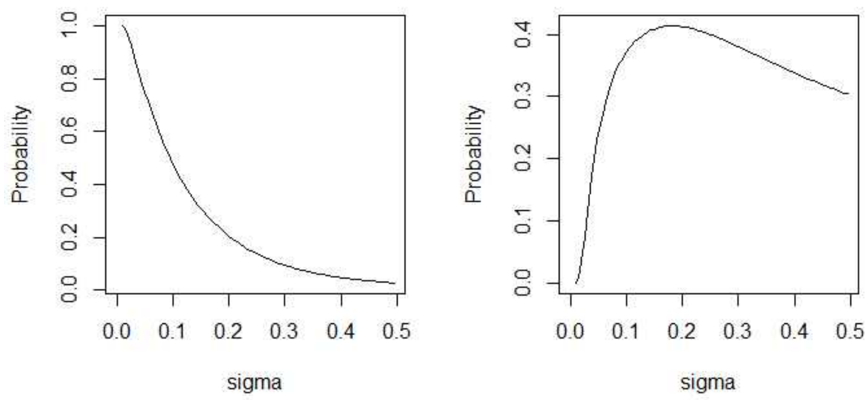


One important reason behind this is that increased σ makes the stock price easier to touch the barrier so that UOC loses effect. On the other hand, price of UOP, which also is an up-and-out option, has lower speed of increase than UIP but still shows a positive slope. A possible explanation could be found in the main difference between UOC and UOP. UOC has two conflicting conditions to be met, in order for the option to pay. The underlying asset has to be larger than pre-specified strike price and it should not be any larger than step barriers for the whole lifetime. On the other hand, barrier conditions and payoff condition at maturity for UOP are in the same direction; underlying asset of UOP should be always smaller than the up-and-out barrier for the entire lifetime and it should also be smaller than the strike price at maturity. Therefore, even when the volatility of the underlying asset is high, the process or the path of the stock contributes to UOP satisfying the payoff conditions.

<Figure 4-6> Icicled step barrier option prices (up=barrier)
under different levels of log-return volatility, with K=100, r=3%



<Figure 4-7> Risk-neutral probability of non-zero payoff with K=100, r=3%,



(a) UOC (b) UOP

<Figure 4-7> shows risk-neutral probability of each option to pay at its maturity, for UOC and UOP respectively. To be specific, the probability functions for (a) and (b) are as following, assuming $k < x_3$.

$$(a)\Pr(X(t_1) \le x_1, X(t_2) \le x_2, X(t_3) \le k, M(0,t_1) \le m_1, M(t_1,t_2) \le m_2, M(t_2,t_3) \le m_3)$$

$$(b)\Pr(X(t_1) \le x_1, X(t_2) \le x_2, k \le X(t_3) \le x_2, M(0,t_1) \le m_1, M(t_1,t_2) \le m_2, M(t_2,t_3) \le m_3)$$

We can note that the probability for UOC drops as the volatility increases, while the probability for UOP rises until certain level and then falls afterwards, as the volatility level goes up. We can infer that this probability affect the price sensitivity with respect to volatility, interacting with innate positive correlation between option price and stock return volatility.

Chapter 5. CONCLUSION

On the basis of previous research on pricing of step barrier options and its icicled variations, this research aimed to provide diverse numerical results and conduct a sensitivity analysis on them. Either as a direct investment product or embedded in equity linked products, this new type of step barrier option can be used widely in products.

This research presents prices of step barrier options and icicled step barrier options with different values of parameters. From step barrier option prices, we could infer how the levels of step barriers affect the option prices. For icicled step barrier options, the numerical results enable readers to compare option prices with different levels of barriers and icicles, risk-free rate and volatility. Especially for risk-free rate and volatility, this paper has conducted sensitivity analysis, in order to clarify multiple consequences the parameter values have on option prices.

Analyzing sensitivity of option price with respect to its parameters is an inevitable step for companies which strive to sell or deal with option embedded products. From this research, due to existence of barriers and icicles, we have discovered the new type of option exhibits sensitivities different to ordinary options. Therefore, when it comes to barrier options or their variations, careful scrutiny on option sensitivities should be carried out, with its innate or unique properties taken into account.

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논문요약

고드름 스텝배리어옵션 (Icicled step barrier option) 가격의 민감도 분석

김은채
보험계리학과
성균관대학교

본 연구는 스텝배리어옵션(step barrier option)과, 이 옵션에 고드름(icicle)이라는 수직 배리어가 추가된 형태의 옵션의 가격에 대해 민감도 분석을 진행한다. Lee, Ko, and Song (2018)에서 처음 고안된 이 옵션은 계약기간을 여러 구간으로 나누어 서로 높이가 다른 배리어를 가지는 스텝배리어옵션에, 각 구간의 종료시점에 수직적인 배리어가 추가된 형태이다. 해당 옵션은 녹인(knock-in), 녹아웃(knock-out) 조건을 가지는 주가연계상품 설계에 유용하게 쓰일 수 있다. 단, 옵션이 내재된 주가연계상품을 판매할 경우 리스크의 헷징(hedging)이 필수적이며, 이를 위해 시장조건이 옵션의 가격에 미치는 영향을 파악하는 것이 필요하다. 따라서 이 연구는 Lee et al. (2018)에 제시된 가격결정 공식을 이용하여 다양한 시장조건, 상품조건 하에서 옵션의 가격을 수치분석하고, 시장 조건에 대한 가격의 민감도를 분석하고자 한다.

주제어 : Barrier Option, Step Barrier Option, Icicled Barrier Option, Sensitivity Analysis