EM FORMULA SHEET CLASS TEST 1

- Product rule: $\frac{d}{dx}\left(f(x)\,g(x)\right) = \frac{df(x)}{dx}\,g(x) + f(x)\,\frac{dg(x)}{dx}$.
- Chain rule: if $r = (x^2 + y^2 + z^2)^{\frac{1}{2}}$, then
$$\frac{\partial}{\partial x}\,f(r) = \frac{df(r)}{dr}\,\frac{\partial r}{\partial x}.$$
- The unit vectors in the x, y and z directions are
$$\vec{e}_x = (1,0,0), \quad \vec{e}_y = (0,1,0), \quad \vec{e}_z = (0,0,1).$$
- A vector $\vec{A}$ with components (A_x, A_y, A_z) can be expressed in terms of the unit vectors $\vec{e}_x, \vec{e}_y$ and $\vec{e}_z$ as
$$\vec{A} = A_x\,\vec{e}_x + A_y\,\vec{e}_y + A_z\,\vec{e}_z.$$
- $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$.
- $\vec{A} \times \vec{B} = (A_y B_z - A_z B_y)\vec{e}_x + (A_z B_x - A_x B_z)\vec{e}_y + (A_x B_y - A_y B_x)\vec{e}_z$.
- The point with Cartesian coordinates (x, y, z) has a position vector
$$\vec{r} = x\,\vec{e}_x + y\,\vec{e}_y + z\,\vec{e}_z.$$
The length of the position vector is
$$r = |\vec{r}\,| = (x^2 + y^2 + z^2)^{\frac{1}{2}}.$$
- The unit vector in the radial direction is
$$\vec{e}_r = \frac{\vec{r}}{r}.$$
- $\vec{\nabla} = (\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z})$.
- The gradient of a scalar field $\phi(\vec{r}\,)$ is the vector field
$$\vec{\nabla}\,\phi(\vec{r}\,) = \left(\frac{\partial \phi}{\partial x}, \frac{\partial \phi}{\partial y}, \frac{\partial \phi}{\partial z}\right).$$
- The divergence of a vector field $\vec{A}(\vec{r}\,)$ is the scalar field
$$\vec{\nabla} \cdot \vec{A}(\vec{r}\,) = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}.$$
- The curl of a a vector field $\vec{A}(\vec{r}\,)$ is the vector field
$$\vec{\nabla} \times \vec{A}(\vec{r}\,) = (\partial_y A_z - \partial_z A_y,\, \partial_z A_x - \partial_x A_z,\, \partial_x A_y - \partial_y A_x)\,.$$

- $$\phi(\vec{r}+d\vec{r}) = \phi(\vec{r}) + \vec{\nabla}\phi(\vec{r}) \cdot d\vec{r}.$$

- $$\begin{aligned} \vec{\nabla} \times \vec{\nabla}\phi(\vec{r},t) &= 0 \\ \vec{\nabla} \cdot (\vec{\nabla} \times \vec{A}(\vec{r},t)) &= 0. \end{aligned}$$

- Line integrals: if Γ is a curve from $\vec{r}_1$ to $\vec{r}_2$, then
$$\int_\Gamma \vec{\nabla}\phi(\vec{r}) \cdot d\vec{\ell} = \phi(\vec{r}_2) - \phi(\vec{r}_1)$$

- Stokes' theorem says that if S is any two-dimensional surface whose boundary is the closed curve Γ, then the circulation of a vector field $\vec{A}(\vec{r})$ around Γ is related to the flux of the curl of the vector field through the surface S:
$$\oint_\Gamma \vec{A}(\vec{r}) \cdot d\vec{\ell} = \int_S \left(\vec{\nabla} \times \vec{A}(\vec{r})\right) \cdot d\vec{S}$$

- Gauss's theorem relates the flux of a vector field through a closed two-dimensional surface S to the integral of the divergence of the vector field over the volume V enclosed by the surface:
$$\oint_S \vec{A}(\vec{r}) \cdot d\vec{S} = \int_V \vec{\nabla} \cdot \vec{A}(\vec{r})\, d^3\vec{r}$$

- Gauss's law: if V is a volume enclosed by a closed surface S, then
$$\oint_S \vec{E}(\vec{r}) \cdot d\vec{S} = \frac{Q}{\epsilon_0},$$
where Q is the charge in the volume V.

- Maxwell's equations in the case of electrostatics:
$$\begin{aligned} \vec{\nabla} \cdot \vec{E}(\vec{r}) &= \frac{\rho(\vec{r})}{\epsilon_0} \\ \vec{\nabla} \times \vec{E}(\vec{r}) &= 0 \end{aligned}$$

- The electric field due to a point charge q at the origin is
$$\vec{E}(\vec{r}) = \frac{q}{4\pi\epsilon_0 r^2}\, \vec{e}_r,$$
where $\vec{e}_r = \frac{\vec{r}}{r}$ is the unit vector in the radial direction, with components $(\frac{x}{r}, \frac{y}{r}, \frac{z}{r})$.

- The electric potential due to a point charge q at the origin is
$$V(\vec{r}) = \frac{q}{4\pi\epsilon_0 r}.$$

- Integral version of Gauss's law: if V is a volume enclosed by a closed surface S, then

$$\oint_S \vec{E}(\vec{r}) \cdot d\vec{S} = \frac{Q}{\epsilon_0},$$

  where Q is the charge in the volume V.

- For static electric fields, the electric potential $V(\vec{r})$ and the electric field $\vec{E}(\vec{r})$ are related as follows:

$$\vec{E}(\vec{r}) = -\vec{\nabla} V(\vec{r}).$$

  If Γ is any path from point $\vec{r}_1$ to point $\vec{r}_2$,

$$V(\vec{r}_2) - V(\vec{r}_1) = -\int_\Gamma \vec{E}(\vec{r}) \cdot d\vec{\ell}.$$