Games

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Recap

Last week we looked at how individuals process new information to update their beliefs about a given process – Bayes' rule.

We also looked at a very simple model of search and briefly discussed the implications to reservation value

- Cost of search
- Risk aversion

We also briefly discussed the concept of bounded rationality and Simon's concept of satisficing.

Games

Today we are going to have a look at behavior in strategic settings.

We will start by looking at a particular class of games, *zero-sum* games

We will then move to a broader class of games and look at the fundamental building block of non-cooperative game theory: the Nash equilibrium.

- We will focus our attention at games where there is more than one equilibrium.

The basics

A game is defined by three basic elements:

- A set of players: N
- A set of strategies: S
- A rule (or function), F, that maps strategies to outcomes.

Game theory's objective is to analyze stable outcomes of a game, given a set of preferences.

Zero-sum (or Strictly Competitive) Games

Early applications of game theory focused on games in which the total gains of all players on any given outcome would add up to zero.

These games are particularly useful to model conflict situations, where both parties cannot mutually agree on a satisfactory outcome.

		Player 2	
		Left	Right
Player 1	Top	A, -A	-B, B
	Down	-C, C	D, -D

Zero-Sum Games

		Player 2	
		Left	Right
Player 1	Top	A, -A	-B, B
	Down	-C, C	D, -D

Assume $A, B, C, D > 0$. Notice that in this case, neither Player 1 nor Player 2 have a preferred strategy:

Zero-Sum Games

		Player 2	
		Left	Right
Player 1	Top	A, -A	-B, B
	Down	-C, C	D, -D

Assume $A, B, C, D > 0$. Notice that in this case, neither Player 1 nor Player 2 have a preferred strategy:

- If Player 1 picks Top with certainty, the best reply by Player 2 is to play Right
- If in turn Player 2 picks Bottom with certainty, the best reply by Player 2 is to play Left

Zero-Sum Games and the Minimax Theorem

In 1928, John von Neumann had a breakthrough in the analysis of zero-sum games when he proved that any zero-sum game with finitely many strategies has a *value*.

That is, Player 1 can come up with a strategy that guarantees him/her an expected payoff of V regardless of what Player 2 does, and vice versa.

In other words, both players can define a strategy that minimizes the maximum payoff the other player can achieve.

Zero-Sum Games, the Minimax Theorem and Mixed Strategy Nash Equilibrium

As it turns out, in Zero-Sum Games, Minimax strategies coincide with mixed strategy Nash equilibrium strategies

- Mixed strategy Nash equilibria are easier to compute, so we'll focus on those

To find the equilibrium of a zero-sum game, we must ask each player to assign a probability to each action available to him/her.

- Let p be Pr(Top), $1 - p$ be Pr(Bottom)
- Let q be Pr(Left), $1 - q$ be Pr(Right)

Zero-Sum Games, the Minimax Theorem and Mixed Strategy Nash Equilibrium

Player 1 will choose p to make Player 2 indifferent between playing Left and Right:

- $E(Left) = p \times (-A) + (1-p) \times (C)$
- $E(Right) = p \times (B) + (1-p) \times (-D)$

- $E(Left) = E(Right) \iff p \times (-A) + (1-p) \times (C) = p \times (B) + (1-p) \times (-D)$
- $p = \frac{C+D}{A+B+C+D}$

Zero-Sum Games, the Minimax Theorem and Mixed Strategy Nash Equilibrium

Player 2 will choose q to make Player 1 indifferent between playing Top and Bottom:

- $E(Top) = q \times (A) + (1 - q) \times (-B)$
- $E(Bottom) = q \times (-C) + (1 - q) \times (D)$

- $E(Top) = E(Bottom) \iff q \times (A) + (1 - q) \times (-B) = q \times (-C) + (1 - q) \times (D)$
- $p = \frac{B+D}{A+B+C+D}$

An example: The Matching Pennies Game

Player 2

		Heads	Tails
Player 1	Heads	1, -1	-1, 1
	Tails	-1, 1	1, -1

Rules:

- Both players simultaneously name one side of a coin
- Player 1 wins if both players name the same side of the coin
- Player 2 wins if the two players name different sides

An example: The Matching Pennies Game

The equilibrium of the MP game is for both players to pick each side of the coin with equal probability.

- That is, $p = 1/2$ and $q = 1/2$.

The value of the MP game is the expected payoff each player gets by playing the equilibrium strategy:

- $V = 0$

Empirical Evidence on Zero-Sum Games: O'Neill (1987)

O'Neill (1987) proposed a very simple experiment to test the theory of zero-sum games.

- The experiment was played by 50 students working in 25 pairs
- Subjects sat opposite each other at a table (no anonymity)
- Each subject held four cards: Ace, 2, 3 and a Joker.
- Each player started the experiment with $2.50 in 5 cent coins.

Empirical Evidence on Zero-Sum Games: O'Neill (1987)

Each round of the experiment worked as follows:

- When prompted, subjects picked a card and placed it face down on the table.
- Following another prompt, subjects then turned the cards over and determined the winner.
- The winner collected 5 cents from the loser, and they moved to the next round.

Payoffs in the O'Neill game

The following matrix displays the payoffs to Player 1 (row player). The payoffs to Player 2 are the negative of the payoff for player 1.

Player 2

		Joker	Ace	Two	Three
Player 1	Joker	+5	-5	-5	-5
	Ace	-5	-5	+5	+5
	Two	-5	+5	-5	+5
	Three	-5	+5	+5	-5

Payoffs in the O'Neill game

Player 1 \ Player 2	Joker	Ace	Two	Three
Joker	+5	-5	-5	-5
Ace	-5	-5	+5	+5
Two	-5	+5	-5	+5
Three	-5	+5	+5	-5

Player 1 wins if both players pick the Joker or if they both pick different numbered cards (the Ace counts as a 1)

Player 2 wins if both players pick the same numbered card or if Player 1 picks the Joker and Player 2 picks any other card.

Equilibrium strategy in the O'Neill game

			Player 2			
			Joker	Ace	Two	Three
			0.4	0.2	0.2	0.2
	Joker	0.4				
Player 1	Ace	0.2				
	Two	0.2				
	Three	0.2				

Note that the equilibrium of this game is invariant to preferences

- There are only two outcomes in this game: a win or a loss...
- ... and gains are preferred to losses.

Note also the clever use of the Joker strategy, which ensures that the equilibrium strategy is asymmetric

Equilibrium strategy in the O'Neill game

TABLE I

RELATIVE FREQUENCIES OF CARD CHOICES IN O'NEILL'S EXPERIMENT[a]

		Column Player Choice				Marginal Frequencies For Row Player:
		1	2	3	*J*	
	1	.044 (.040) [.004]	.043 (.040) [.004]	.043 (.040) [.004]	.091 (.080) [.005]	.221 (.200) [.008]
	2	.046 (.040) [.004]	.038 (.040) [.004]	.038 (.040) [.004]	.092 (.080) [.005]	.215 (.200) [.008]
Row Player Choice	3	.049 (.040) [.004]	.032 (.040) [.004]	.037 (.040) [.004]	.085 (.080) [.005]	.203 (.200) [.008]
	J	.086 (.080) [.005]	.065 (.080) [.005]	.051 (.080) [.005]	.158 (.160) [.007]	.362 (.400) [.010]
Marginal Frequencies for Column Player:		.226 (.200) [.008]	.179 (.200) [.008]	.169 (.200) [.008]	.426 (.400) [.010]	

[a] Numbers in parentheses represent minimax predicted relative frequencies. Numbers in brackets represent standard deviations for observed relative frequencies under the minimax hypothesis.

Behavior in the O'Neill game

The choice frequencies in the experiment match closely those of theory.

- Player 1s chose the Joker 36.2% of the time $(0.362 = 0.400, t = 0.051)$
- Player 2s chose the Joker 43.0% of the time $(0.430 = 0.400, t = 0.150)$

Behavior in the O'Neill game

O'Neill also argues that the frequency of choices of numbered cards (Ace, Two and Three) is pretty close to theory:

- Player 1s chose 578 aces, 565 twos and 532 threes
- Player 2s chose 593 aces, 470 twos and 446 threes
- Only the P2s distribution was significantly different from the prediction of equal probabilities using a χ^2 test.
- O'Neill argued that this was probably due to the Ace being a "loaded" card.

Behavior in the O'Neill game

O'Neill finally looks at winning probabilities in the data.

The theory predicts that:

- The row player should win 40% of the time
- The column player should win 60% of the time

The data:

- The row players won 40.1% of the time
- The column players won 59.9% of the time

Lies, Damn Lies and Statistics: Brown and Rosenthal (1990)

While the data looks close to Minimax play, Brown and Rosenthal (1990) re-evaluate O'Neill's data, and show that in fact Minimax can be confidently ruled out.

There are two issues to consider. The first is that although the choice frequencies are close to prediction, they may be based on a very different decision model.

For example, suppose Player 1 believes Player 2 thinks Player 1 picks a card at random 20% of the time. The other 80% of the time, Player 1 actually randomizes.

Lies, Damn Lies and Statistics

Let's work out the optimal behavior for Player 2 if (s)he believes Player 1 is randomizing:

$$E_2(Joker) = \frac{1}{4}(-5+5+5+5) = 4$$
$$E_2(Ace) = \frac{1}{4}(+5+5-5-5) = 0$$
$$E_2(Two) = \frac{1}{4}(+5-5+5-5) = 0$$
$$E_2(Three) = \frac{1}{4}(+5-5-5+5) = 0$$

In short, if Player 2 believes Player 1 picks cards at random, (s)he would always pick the Joker card.

So, what would Player 1 do if (s)he believes Player 2 is picking the Joker card?

Lies, Damn Lies and Statistics

So, if Player 1 believes Player 2 thinks Player 1 picks a card at random 20% of the time, he will pick the Joker 20% of time.

If Player 1 picks at random the rest of the time, here's what the strategy of Player 1 looks like:

- 0.4, 0.2, 0.2, 0.2 for Joker, Ace, Two and Three.

This is indistinguishable from Minimax!

Lies, Damn Lies and Statistics: testing joint distributions of behavior

So, when testing for the null hypothesis of Minimax play, we need to use more stringent tests than looking at the frequencies of play of a particular card.

Brown and Rosenthal start by running a Chi-Squared test on the joint distribution of play by both sets of players against the predicted Minimax.

- They rejected Minimax play at $p < 0.01$

Lies, Damn Lies and Statistics: winning probabilities are not good measures of behavior

But what about the winning probability? Surely that is the key piece of evidence?
Well, the problem is that a lot of (random) behavior will give very similar probabilities of winning.

- If both players just pick at random, the row player will win with 43% probability!

Lies, Damn Lies and Statistics: Looking at the data pair-by-pair

TABLE II

RELATIVE FREQUENCIES OF CARD CHOICES AND ROW-PLAYER WINS IN O'NEILL'S EXPERIMENT BY PLAYER PAIR

Pair #	Winning % for Row Player	Row Player Choice 1	2	3	J	Column Player Choice 1	2	3	J	Comments
1	.391	.257	.286*	.276	.181*	.229	.210	.210	.352	a, c
2	.295	.171	.181	.210	.438	.152	.152	.143	.552*	b
3	.390	.162	.114*	.181	.543*	.219	.133	.095*	.552*	a, b, c
4	.419	.219	.267	.181	.333	.067*	.086*	.124	.724*	b, c
5	.343	.171	.171	.190	.467	.324*	.086*	.143	.448	b, c
6	.419	.257	.143	.210	.390	.257	.200	.095*	.448	b
7	.476	.238	.229	.229	.305*	.219	.190	.238	.352	—
8	.467	.171	.286*	.219	.324	.267	.248	.190	.295*	—
9	.362	.181	.257	.267	.295*	.257	.181	.219	.343	—
10	.390	.257	.171	.152	.419	.200	.190	.200	.410	—
11	.390	.276	.248	.171	.305*	.229	.200	.200	.371	—
12	.543	.276	.133	.105*	.486	.210	.200	.162	.429	a, c
13	.410	.219	.267	.248	.267*	.114*	.219	.133	.533*	a, b, c
14	.467	.267	.229	.200	.305*	.267	.248	.257	.229*	b, c
15	.324	.200	.181	.162	.457	.295*	.143	.190	.371	—
16	.343	.152	.248	.162	.438	.219	.238	.162	.381	—
17	.362	.200	.219	.219	.362	.229	.171	.190	.410	—
18	.486	.238	.181	.190	.390	.219	.152	.219	.410	—
19	.390	.286*	.190	.200	.324	.171	.162	.162	.505	—
20	.438	.210	.219	.143	.429	.210	.171	.124	.495	—
21	.476	.190	.229	.210	.371	.276	.219	.181	.324	—
22	.400	.200	.162	.181	.457	.286*	.181	.190	.343	—
23	.448	.229	.286*	.324*	.162*	.295*	.181	.105*	.419	a, b, c
24	.495	.248	.257	.238	.257*	.248	.162	.219	.371	a
25	.333	.238	.229	.200	.333	.181	.152	.076*	.590*	b, c

* Denotes rejection (at .05 level) of minimax binomial model for a given card.

[a] Denotes joint rejection (at .05 level) of minimax multinomial model for all cards chosen by the row player, based on Pearson statistic and $\chi^2(3)$.

[b] Denotes joint rejection (at .05 level) of minimax multinomial model for all cards chosen by the column player, based on Pearson statistic and $\chi^2(3)$.

[c] Denotes joint rejection (at .05 level) of minimax multinomial model for all cards chosen by both players, based on Pearson statistic and $\chi^2(6)$.

Lies, Damn Lies and Statistics: Looking at the data individual-by-individual

TABLE IV

RESULTS OF SIGNIFICANCE TESTS FROM LOGIT EQUATIONS FOR THE CHOICE OF A JOKER CARD

Estimating Equation[a]: $J = G[a_0 + a_1 \text{lag}(J) + a_2 \text{lag2}(J) + b_0 J^* + b_1 \text{lag}(J^*) + b_2 \text{lag2}(J^*) + c_1 \text{lag}(J)\text{lag}(J^*) + c_2 \text{lag2}(J)\text{lag2}(J^*)]$

	Null Hypothesis	Player Pairs Whose Behavior Allows Rejection of the Null Hypothesis at the .05 Level
(1)	a_1, a_2, b_0, b_1, b_2, c_1, c_2 all $= 0$	Row: 2, 5, 7, 8, 10, 11, 12, 14, 16, 17, 20, 21, 22 Column: 2, 4, 6, 7, 8, 9, 10, 11, 12, 14, 17, 18, 19, 20, 21, 23, 24, 25
(2)	$a_1, a_2 = 0$	Row: 6, 7, 8, 10, 12, 17, 21, 22 Column: 4, 7, 8, 9, 10, 11, 12, 13, 14, 15, 17, 19, 20, 23, 24, 25
(3)	b_1, b_2, c_1, c_2 all $= 0$	Row: 4, 5, 7, 8, 10, 11, 12, 14, 16, 17, 21, 22, 23, 25 Column: 1, 2, 17, 18, 19, 21, 24, 25
(4)	$c_1, c_2 = 0$	Row: 8, 9, 10, 11, 12, 14, 25 Column: 2, 6, 9, 17, 21, 24, 25
(5)	$b_1, b_2 = 0$	Row: 4, 5, 7, 10, 12, 14, 16, 17, 21, 23, 25 Column: 1, 2, 17, 19, 21, 25
(6)	$b_0 = 0$	Row: 2, 4 Column: 2, 4

[a] The symbols J and J^* denote the choice of a joker card by a player and by his opponent, respectively. The function $G[x]$ denotes the function $\exp(x)/[1 + \exp(x)]$. Rejections are based on likelihood-ratio tests.

Testing for Randomness using repeated measures

This experiment is a very good example of the methodological problems faced by experimental economists:

- Subjects require repetition in order to learn how to play the game.
- But repetition introduces dynamic effects, particularly if you are playing the same person/people every time.
- One-shot decisions may not always help, since in the O'Neill game, what you want to measure is a probability (1 obs per subject doesn't give you much power)

Alternative: Use Experts!

Professionals Play Minimax

Palacios-Huerta (2003) studies 1417 penalty kicks taken in professional games in European competitions.

TABLE 1

Distribution of strategies and scoring rates

Score difference	#Obs.	LL	LC	LR	CL	CC	CR	RL	RC	RR	Scoring rate
0	580	16·9	1·3	21·0	4·3	0·8	5·6	19·4	0·6	27·9	81·9
1	235	19·1	0	19·1	4·2	0	2·5	28·0	0	26·8	77·8
−1	314	19·7	0·9	25·8	1·9	0	6·4	20·0	0·6	30·2	80·2
2	97	23·7	2·0	17·5	5·2	0	0	20·6	1·0	29·9	75·2
−2	114	26·3	0	25·4	3·5	0	3·5	16·6	0	24·5	78·0
3	27	14·8	0	18·5	3·7	0	11·1	22·2	0	29·6	77·7
−3	23	30·4	0	30·4	0	0	0	21·7	0	17·4	82·6
4	7	42·8	0	28·5	0	0	0	14·2	0	14·2	100
−4	12	25·0	0	25·0	0	0	16·6	16·6	0	16·6	83·3
Others	8	50·0	0	0	0	0	12·5	37·5	0	0	87·5
Penalties shot in:											
First half	558	21·1	0·8	19·8	3·9	0·3	3·5	20·0	0·3	29·7	82·9
Second half	859	18·7	0·9	23·2	3·3	0·3	3·6	22·8	0·5	26·3	78·3
Last 10 min	266	21·8	0	21·0	0·3	0	0·7	25·1	0	30·8	73·3
All penalties	1417	19·6	0·9	21·9	3·6	0·3	3·6	21·7	0·5	27·6	80·1
Scoring rate	80·1	55·2	100·0	94·2	94·1	50·0	82·3	96·4	100·0	71·1	

Note: The first letter of the strategy denotes the kicker's choice and the second the goalkeeper's choice. "R" denotes the

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TABLE 2

Distribution of strategies and scoring rates by kicker type

Score difference	#Obs.	LL	LC	LR	CL	CC	CR	RL	RC	RR	Scoring rate
		Left-footed kickers									
0	174	17·8	1·7	20·1	6·3	0	8·6	22·9	0·5	21·8	82·7
1	73	28·7	0	30·1	4·1	0	2·7	19·1	0	15·0	78·0
−1	92	29·3	1·0	26·0	1·0	0	2·0	21·7	1·0	18·4	82·6
2	29	51·7	0	13·7	3·0	0	0	10·3	0	20·6	72·4
−2	30	40·0	0	13·3	3·0	0	3·0	20·0	0	20·0	76·6
All penalties	406	29·3	1·4	20·4	4·4	0	3·9	23·8	0	16·5	
Scoring rate	81·0	62·1	100	95·1	94·4	0	81·2	93·8	0	61·2	
		Right-footed kickers									
0	406	16·4	1·2	21·4	3·4	1·2	4·4	20·4	0·7	30·5	83·2
1	162	14·8	0	14·2	4·3	0	2·4	32·1	0	32·1	77·7
−1	222	15·7	1·0	25·6	2·2	0	0	19·3	1·0	35·1	80·6
2	68	11·7	2·9	19·1	5·8	0	0	25·0	1·4	33·8	76·4
−2	84	21·4	0	29·7	3·5	0	3·5	15·4	0	26·2	78·5
All penalties	1011	15·8	0·6	22·5	3·2	0·5	3·4	20·8	0·6	32·1	
Scoring rate	79·8	50·0	100	93·8	93·9	60·0	82·8	97·6	100	73·2	

Note: The first letter of the strategy denotes the kicker's choice and the second the goalkeeper's choice. "R" denotes the R.H.S. of the goalkeeper, "L" denotes the L.H.S. of the goalkeeper, and "C" denotes centre.

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TABLE 3

Tests for equality of scoring probabilities

Player	#Obs.	Mixture L	Mixture R	Scoring rates L	Scoring rates R	Pearson statistic	*p*-value
Kicker 1	34	0·32	0·68	0·91	0·91	0·000	0·970
Kicker 2	31	0·35	0·65	0·82	0·80	0·020	0·902
Kicker 3	40	0·48	0·52	0·74	0·76	0·030	0·855
Kicker 4	38	0·42	0·58	0·88	0·91	0·114	0·735
Kicker 5	38	0·50	0·50	0·79	0·84	0·175	0·676
Kicker 6	36	0·28	0·72	0·70	0·77	0·185	0·667
Kicker 7	41	0·20	0·80	0·75	0·82	0·191	0·662
Kicker 8	35	0·31	0·69	0·82	0·75	0·199	0·656
Kicker 9	31	0·19	0·81	0·83	0·92	0·416	0·519
Kicker 10	35	0·37	0·63	0·86	0·77	0·476	0·490
Kicker 11	32	0·48	0·52	0·87	0·94	0·521	0·471
Kicker 12	32	0·48	0·52	0·87	0·94	0·521	0·471
Kicker 13	38	0·55	0·45	0·76	0·88	0·907	0·341
Kicker 14	30	0·33	0·67	0·90	0·75	0·938	0·333
Kicker 15	30	0·50	0·50	0·80	0·93	1·154	0·283
Kicker 16	42	0·43	0·57	0·89	0·75	1·287	0·257
Kicker 17	40	0·42	0·58	0·58	0·85	1·637	0·201
Kicker 18	46	0·44	0·56	0·90	0·77	1·665	0·197
Kicker 19	39	0·48	0·52	0·74	0·90	1·761	0·184
Kicker 20	40	0·35	0·65	0·93	0·69	2·913	0·088*
Kicker 21	40	0·42	0·58	0·65	0·91	4·322	0·038**
Kicker 22	40	0·40	0·60	1·00	0·75	4·706	0·030**
All kickers	808	0·3998	0·6002	0·8111	0·8268		
Goalkeeper 1	37	0·38	0·62	0·21	0·22	0·000	0·982
Goalkeeper 2	38	0·39	0·61	0·20	0·22	0·017	0·898
Goalkeeper 3	30	0·60	0·40	0·28	0·25	0·028	0·866
Goalkeeper 4	50	0·46	0·54	0·17	0·15	0·061	0·804
Goalkeeper 5	36	0·33	0·67	0·25	0·21	0·080	0·777
Goalkeeper 6	34	0·44	0·56	0·27	0·21	0·147	0·702
Goalkeeper 7	37	0·19	0·81	0·14	0·10	0·221	0·638
Goalkeeper 8	37	0·54	0·46	0·25	0·18	0·293	0·588
Goalkeeper 9	32	0·56	0·44	0·22	0·14	0·326	0·568
Goalkeeper 10	40	0·45	0·55	0·11	0·18	0·388	0·533
Goalkeeper 11	33	0·18	0·82	0·17	0·30	0·416	0·519
Goalkeeper 12	30	0·27	0·73	0·25	0·14	0·545	0·460
Goalkeeper 13	34	0·41	0·59	0·14	0·25	0·578	0·447
Goalkeeper 14	40	0·50	0·50	0·15	0·25	0·625	0·429
Goalkeeper 15	44	0·45	0·55	0·10	0·21	0·957	0·328
Goalkeeper 16	36	0·31	0·69	0·09	0·24	1·804	0·298
Goalkeeper 17	42	0·55	0·45	0·30	0·11	2·449	0·118
Goalkeeper 18	42	0·38	0·62	0·13	0·35	2·506	0·113
Goalkeeper 19	42	0·40	0·60	0·35	0·12	3·261	0·071*
Goalkeeper 20	40	0·60	0·40	0·08	0·37	5·104	0·024**
All goalkeepers	754	0·4231	0·5769	0·1943	0·2068		

Note: *Indicates rejected at 10% level, and **indicates rejected at 5% level.

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TABLE 5

Tests of serial independence of choices

Player	Observations L	Observations R	Observations Total	Runs R	$\Phi[f(r-1;s)]$	$\Phi[f(r;s)]$
Kicker 1	11	23	34	16	0·439	0·597
Kicker 2	11	20	31	21	0·983**	0·994
Kicker 3	19	21	40	22	0·570	0·691
Kicker 4	16	22	38	19	0·365	0·496
Kicker 5	19	19	38	22	0·689	0·795
Kicker 6	10	26	36	15	0·344	0·509
Kicker 7	8	33	41	14	0·423	0·625
Kicker 8	11	24	35	15	0·263	0·407
Kicker 9	6	25	31	9	0·097	0·241
Kicker 10	13	22	35	19	0·599	0·729
Kicker 11	15	17	32	19	0·714	0·822
Kicker 12	15	17	32	20	0·822	0·901
Kicker 13	21	17	38	23	0·816	0·891
Kicker 14	10	20	30	12	0·117	0·221
Kicker 15	15	15	30	18	0·711	0·824
Kicker 16	18	24	42	19	0·164	0·254
Kicker 17	19	21	40	20	0·321	0·443
Kicker 18	20	26	46	19	0·693	0·789
Kicker 19	19	20	39	19	0·259	0·374
Kicker 20	14	26	40	14	0·022	0·049*
Kicker 21	17	23	40	18	0·159	0·251
Kicker 22	16	24	40	22	0·668	0·779
Goalkeeper 1	14	23	37	17	0·249	0·374
Goalkeeper 2	15	23	38	21	0·678	0·790
Goalkeeper 3	18	12	30	12	0·065	0·130
Goalkeeper 4	23	27	50	24	0·250	0·350
Goalkeeper 5	12	24	36	17	0·424	0·576
Goalkeeper 6	15	19	34	15	0·124	0·212
Goalkeeper 7	7	30	37	13	0·533	0·738
Goalkeeper 8	20	17	37	20	0·516	0·647
Goalkeeper 9	18	14	32	19	0·739	0·842
Goalkeeper 10	18	22	40	14	0·009	0·021**
Goalkeeper 11	6	27	33	11	0·423	0·661
Goalkeeper 12	8	22	30	15	0·802	0·908
Goalkeeper 13	14	20	34	19	0·644	0·767
Goalkeeper 14	20	20	40	22	0·564	0·685
Goalkeeper 15	20	24	44	27	0·871	0·925
Goalkeeper 16	11	25	36	16	0·378	0·535
Goalkeeper 17	23	19	42	28	0·964*	0·983
Goalkeeper 18	16	26	42	23	0·713	0·814
Goalkeeper 19	17	25	42	18	0·113	0·187
Goalkeeper 20	24	16	40	19	0·285	0·408

Note: *Indicates rejected at 10% level, and **indicates rejected at 5% level.

Professionals Play Minimax

TABLE 6

Results of significance tests from logit equations for the choice of the natural side

Estimating equation:

$$R = G[a_0 + a_1\text{lag}(R) + a_2\text{lag2}(R) + b_0R^* + b_1\text{lag}(R^*) + b_2\text{lag2}(R^*) + c_1\text{lag}(R)\text{lag}(R^*) + c_2\text{lag2}(R)\text{lag2}(R^*)]$$

Null hypothesis:		Players whose behaviour allows rejection of the null hypothesis at the: 0·05 level	0·10 level	0·20 level
1. $a_1 = a_2 = b_0 = b_1 = b_2 = c_1 = c_2 = 0$	Kicker	–	2	2, 18
	Goalkeeper	–	7	7, 15
2. $a_1 = a_2 = 0$	Kicker	–	2	2, 14
	Goalkeeper	–	8	8, 17
3. $b_1 = b_2 = 0$	Kicker	–	–	5
	Goalkeeper	–	7	7
4. $c_1 = c_2 = 0$	Kicker	–	–	6
	Goalkeeper	–	–	14
5. $b_0 = 0$	Kicker	–	11, 17	5, 11, 17, 21
	Goalkeeper	–	3, 16	3, 9, 10, 16

Notes: R and R^* denote the choice of "natural" strategy by a kicker and a goalkeeper, respectively (right for a right-footed kicker and for a goalkeeper facing a right-footed kicker, and left for a left-footed kicker and for a goalkeeper facing a left-footed kicker). The terms "lag" and "lag2" refer to the strategies previously followed in the ordered sequence of penalty kicks. $G[x]$ denotes the function $\exp(x)/[1+\exp(x)]$. Rejections are based on likelihood-ratio tests.

Professionals Play Minimax

Experienced players may be less prone to behavioral biases/more rational.

- Higher stakes
- They don't face the same opponent in consecutive penalty kicks

Non-Zero-Sum Games

Game theory is interested in finding outcomes from which players have no incentive to deviate.

- i.e. outcomes in which my actions are optimal given what the other players are doing (and vice versa).

Such an outcome is a Nash Equilibrium (NE) of a game.

Some games have a unique NE; others have many NE.

Example 1

Take two workers operating in a factory.

Their payoff is a function of joint output

However each worker has a private cost of effort

Example 1

$N = \{1, 2\}$

$S_i = \{High, low\}$

		Player 2	
		High	Low
Player 1	High	20, 20	5, 15
	Low	15, 5	10, 10

Example 1

There are two Nash equilibria in pure strategies:

- (High, High)
- (Low, Low)

In this game, strategies are strategic complements:

Player 2's best response to a rise (drop) in player 1's action is a rise (drop) in his action.

Which equilibrium should be played?

Coordination Games

This particular type of game is interesting to economists as it captures the idea of externalities:

- Team production processes (e.g. min. effort game);
- Industrial Organisation (e.g. market entry games);

It is important to understand why would a set of agents be stuck in bad equilibria.

Is this due to strategic or behavioural reasons?

Common criteria for equilibrium selection:

Focal points;

Payoff dominance;

Risk dominance.

Focal points

Thomas Schelling proposed a class exercise to his students.

They had to select a time and a place to meet up in New York city the following day.

The majority of his students chose Grand Central Station at 12 noon.

Certain equilibria are "intuitive" or naturally salient and as a result get chosen more often.

Focal points: Crawford et al. (2008)

This paper studies the extent to which salience of decision labels could lead to resolution of the coordination problem.

In pilot data, they modified Schelling's example and set up a simple coordination game

University of Chicago students had to choose to meet in one of two locations:

- The Sears Tower, a landmark Chicago building;
- The AT&T Tower, a little known building across the street from the Sears Tower.

They considered three conditions

Focal points: Crawford et al. (2008)

Symmetric Treatment:

		Player 2	
		Sears Tower	AT&T Tower
Player 1	Sears Tower	100, 100	0, 0
	AT&T Tower	0, 0	100, 100

Focal points: Crawford et al. (2008)

Slightly Asymmetric Treatment:

		Player 2	
		Sears Tower	AT&T Tower
Player 1	Sears Tower	101, 100	0, 0
	AT&T Tower	0, 0	100, 101

Focal points: Crawford et al. (2008)

Asymmetric Treatment:

		Player 2	
		Sears Tower	AT&T Tower
Player 1	Sears Tower	110, 100	0, 0
	AT&T Tower	0, 0	100, 110

Focal points: Crawford et al. (2008)

The percentage of subjects who chose “Sears Tower” is as follows:

Treatment		High Payoff	Low Payoff
Symmetry	90% (n=60)		
Slight Asymmetry		58% (n=50)	61% (n=49)
Asymmetry		47% (n=30)	50% (n=28)

Focal points: Crawford et al. (2008)

Expected coordination rates were equal to:

- Symmetry: 82%
- Slight Asymmetry: 52%
- Asymmetry: 50%
- Mixed Strategy Nash Equilibrium: 50%!

The mere presence of small payoff asymmetries dramatically reduces the power of focal points (in Crawford et al.'s data set).

Payoff Dominance

Payoff dominance is a relatively intuitive concept;

If an equilibrium is Pareto superior to all other NE, then it is payoff dominant.

An outcome Pareto-dominates another if all players are at least as well off and at least one is strictly better off.

It is intuitively appealing, but the data does not seem to fully support it.

Risk Dominance

The concept of risk dominance is based upon the idea that a particular equilibrium may be riskier than another.

- This is NOT related to concavity of the utility function!!!

In simple 2x2 games, RD could be thought as how costly are deviations from a particular equilibrium vis--vis the other?

- There is no general way to compute a risk dominant equilibrium in $n \times n$ games.

Although rational agents ought to follow payoff dominance, experimental data shows subjects often play the risk dominant ("safer") equilibrium.

Obstacles to coordination

Larger N

The concept of coordination centers around beliefs

The choice of equilibrium will depend on what you think the other player will do.

The more players there are, the harder it is to coordinate: it is harder to form consistent beliefs about every players action — it only takes one player to destroy the equilibrium.

Obstacles to coordination

Incentive structure

Are equilibria unfair?

Are there focal points?

Can players communicate?

Cooper et al. (1992)

Run a simple coordination experiment.

Vary the extent subjects can communicate with one another:

- No communication;
- One-way (non-binding) announcements;
- Two-way (non-binding) announcements.

Cooper et al. (1992)

		Column Player's Strategy	
		1	2
Row Player's Strategy	1	800,800	800,0
	2	0,800	1000,1000

FIGURE II

Cooper et al. (1992)

TABLE I
SCG

		Strategy	
		1	2
Announcements			
No communication:		—	—
One-way:			
Rep. 1&3		19	91
Rep. 2		2	53
Total		21	144
Two-way:		0	330
Actions:			
No communication:		325	5
One-way:			
Rep. 1&3		88	132
Rep. 2		15	95
Total		103	227
Two-way:		15	315
		Action pair	
	(1,1)	(2,2)	(1,2), (2,1)
Treatment:			
No communication:	160	0	5
One-way:			
Rep. 1&3	25	47	38
Rep. 2	1	41	13
Total	26	88	51
Two-way:	0	150	15

Cooper et al. (1992)

Communication works, but particularly if it is 2-sided.

It appears to have a reassurance component in that both players can reassure each other of their intentions regarding each other.