HEAT TRANSFER

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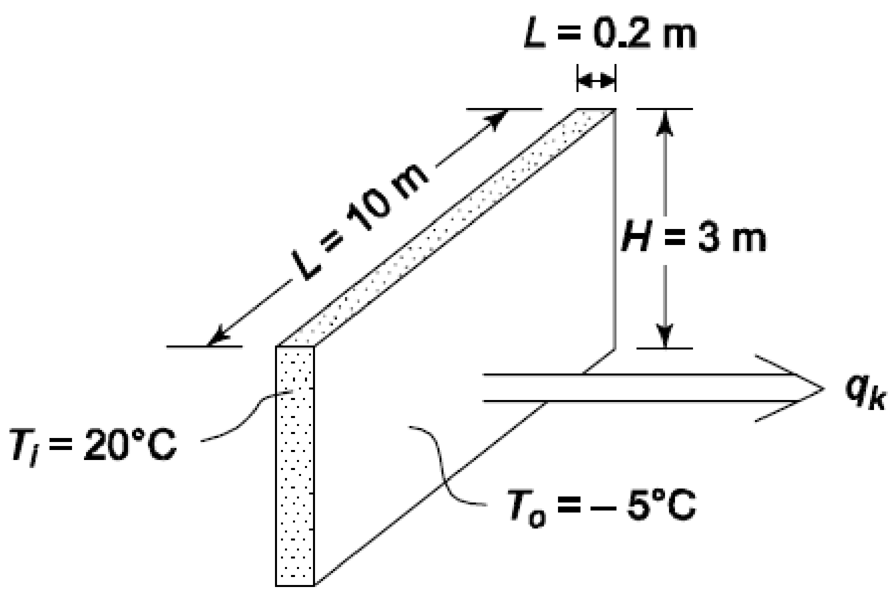
Problems

Problem 1

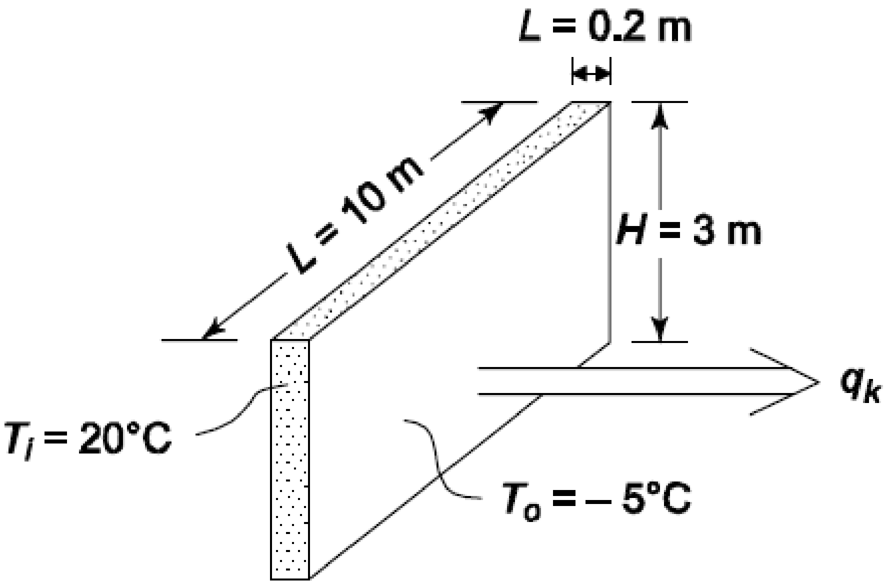
If inner and outer faces of a concrete wall with thickness of 20 cm is kept at a temperature 20 °C and -5 C, respectively, and the thermal conductivity of the concrete is 1.2 W/m.K. Determine the heat loss through a wall 10 m long and 3 m high.



Problems

Problem 1

- 10 m long, 3 m high, and 0.2 m thick concrete wall.
- Thermal conductivity of the concrete (k) = 1.2 W/m.K
- Temperature of the inner surface (T_i) = 20 °C
- Temperature of the outer surface (T_o) = –5 °C
- One dimensional heat flow
- The system has reached steady state.



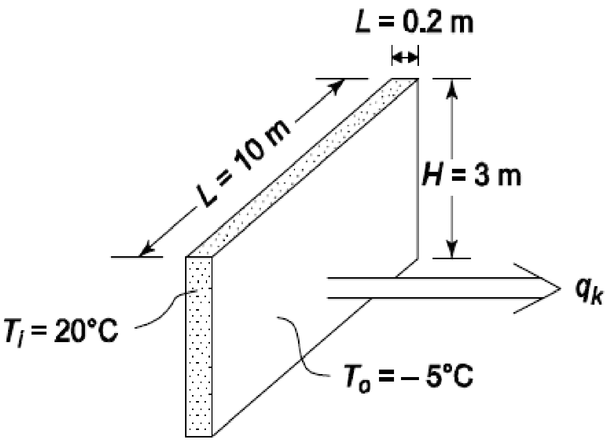
Problems

Problem 1

$$q = \frac{A\,K}{L}(\Delta T)$$

$$q = \frac{(10\,\text{m})(3\,\text{m})\big(1.2\ \text{W/(m K)}\big)}{0.2\,\text{m}}\ (20°\text{C} - (-5°\text{C}))$$

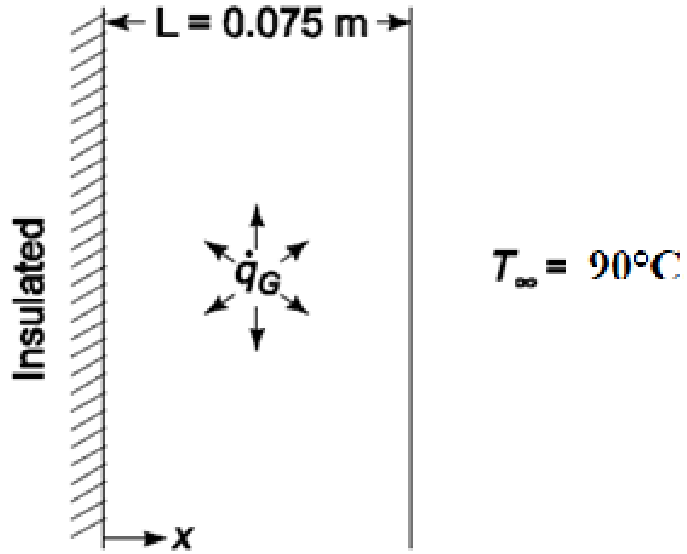
$$q = 4500\ \text{W}$$



Problems

Problem 2

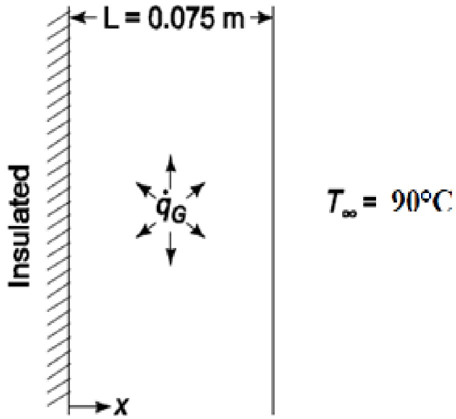
A wall with 7.5 cm thickness (shown below) generates heat at the rate of 105 W/m^3. One side of the wall is insulated, and the other side is exposed to an environment at 90°C. The convective heat transfer coefficient between the wall and the environment is 500 W/m^2.K. Under one-dimensional-steady state conditions, and if the thermal conductivity of the wall is 12 W/m.K, calculate the maximum temperature in the wall.



Problems

Problem 2

- Plane wall with internal heat generation
- Thickness (L) = 0.075 m
- Internal heat generation rate (q_G) = 105 W/m3
- One side is insulated
- Ambient temperature on the other side (T) = 90 °C
- Convective heat transfer coefficient (h_c) = 500 W/(m2 K)
- Thermal conductivity (k) = 12 W/m.K.
- The heat loss through the insulation is negligible.
- The system has reached steady state.
- One dimensional conduction through the wall.



Problems

Problem 2

The one dimensional conduction equation:

$$k \frac{\partial^2 T}{\partial x^2} + \dot{q}_G = \rho c \frac{\partial T}{\partial t}$$

For steady state, $\frac{\partial T}{\partial t} = 0$ therefore

$$k \frac{d^2 T}{d x^2} + \dot{q}_G = 0$$

$$\frac{d^2 T}{d x^2} = -\frac{\dot{q}_G}{k}$$

To solve this equation, two boundary conditions are needed:

1. No energy loss through the insulator.
2. Convection at the other surface

Problems

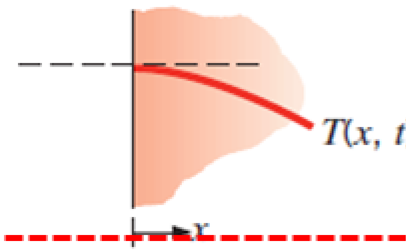
Problem 2

1. No energy loss through the insulator.

Insulated surface

Adiabatic or insulated surface

$$\left.\frac{\partial T}{\partial x}\right|_{x=0} = 0 \qquad (2.33)$$

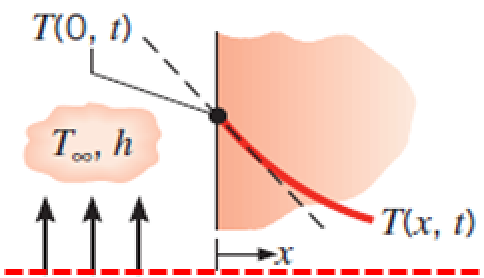


2. Convection at the other surface

Fluid flow near to the surface

Convection surface condition

$$-k\left.\frac{\partial T}{\partial x}\right|_{x=0} = h[T_\infty - T(0, t)] \qquad (2.34)$$



Problems

Problem 2

1. No energy loss through the insulator.

$$\frac{d^2T}{dx^2} = -\frac{\dot{q}_G}{k} \Rightarrow \frac{dT}{dx} = -\frac{\dot{q}_G}{k} x + C_1$$

$$\frac{dT}{dx} = 0 \text{ at } x = 0$$

$$0 = -\frac{\dot{q}_G}{k}(0) + C_1 \Rightarrow C_1 = 0$$

Integrating again

$$T = -\frac{\dot{q}_G}{2k} x^2 + C_2$$

Problems

Problem 2

2. Convection at the other surface

$$\boxed{T = -\frac{\dot{q}_G}{2k}x^2 + C_2}$$

$$-k\frac{d\boxed{T}}{dx} = h_c\left(\boxed{T} - T_\infty\right) \qquad \text{at} \quad x = L$$

$$-k\left(\frac{\dot{q}_G L}{k}\right) = h_c\left(-\frac{\dot{q}_G L^2}{2k} + C_2 - T_\infty\right) \Rightarrow C_2 = \dot{q}_G L\left(\frac{1}{h_c} + \frac{L}{2k}\right) + T_\infty$$

Problems

Problem 2

Substituting this into the expression for T yields the temperature distribution in the wall:

$$T(x) = \frac{\dot{q}_G}{2k}\, x^2 + \dot{q}_G L \left(\frac{1}{\bar{h}_c} + \frac{L}{2k} \right) + T_\infty$$

$$T(x) = T_\infty + \frac{\dot{q}_G}{2k} \left(L^2 + \frac{2kL}{\bar{h}_c} - x^2 \right)$$

The maximum temperature occurs at $x = 0$.

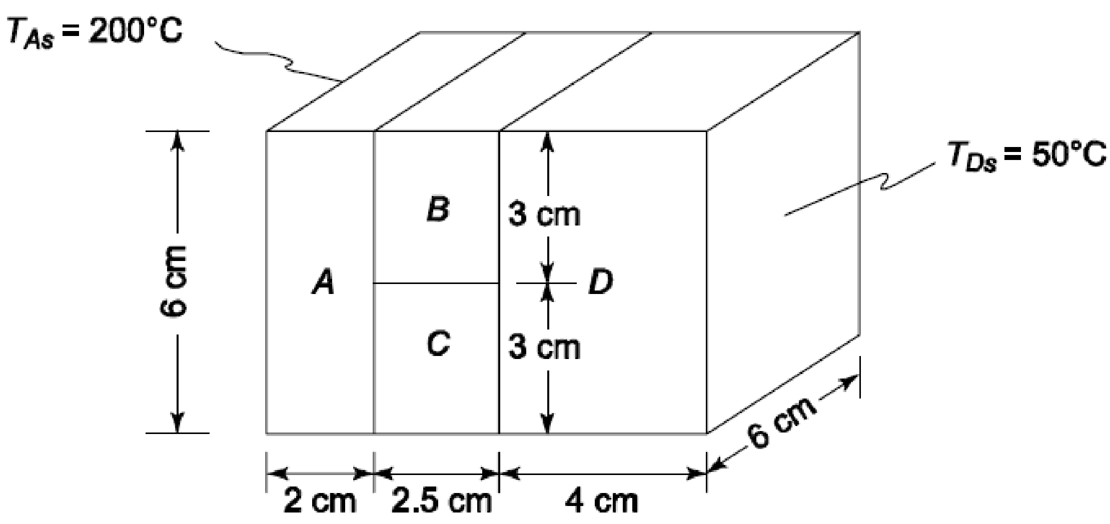
$$T_{\max} = T_\infty + \frac{\dot{q}_G}{2k} \left(L^2 + \frac{2kL}{\bar{h}_c} \right)$$

$$T_{\max} = 90°\mathrm{C} + \frac{10^5\ \mathrm{W/m^3}}{2[12\mathrm{W/(mK)}]} \left((0.075\,\mathrm{m})^2 + \frac{2[12\ \mathrm{W/(mK)}](0.075\,\mathrm{m})}{500\ \mathrm{W/(m^2K)}} \right) = 128°\mathrm{C}$$

Problems

Problem 3

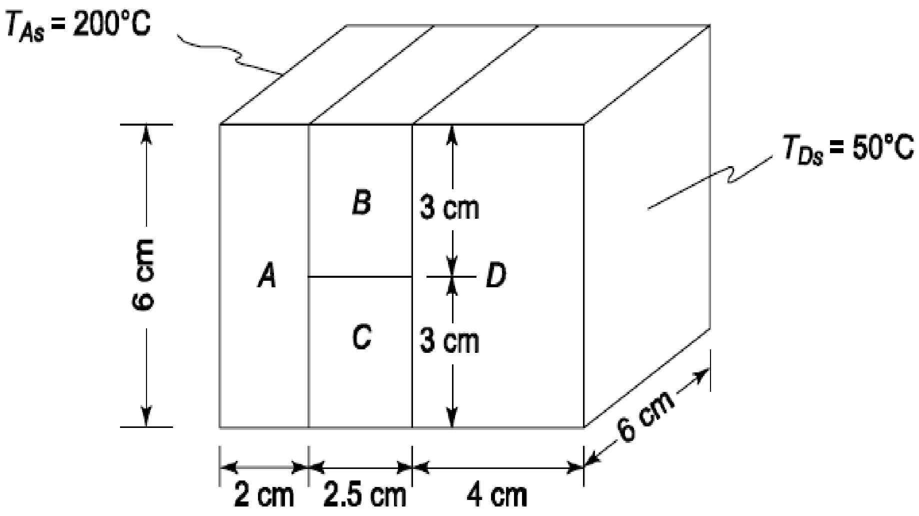
A composite wall (shown below) has uniform temperatures. If the thermal conductivities of the wall materials are: k_A = 70 W/m.K, k_B = 60 W/m.K, k_C = 40 W/m.K, and k_D = 20 W/m.K, determine the rate of heat transfer through this section of the wall and the temperatures at the interfaces. (Surfaces normal to heat transfer direction are isothermal).



Problems

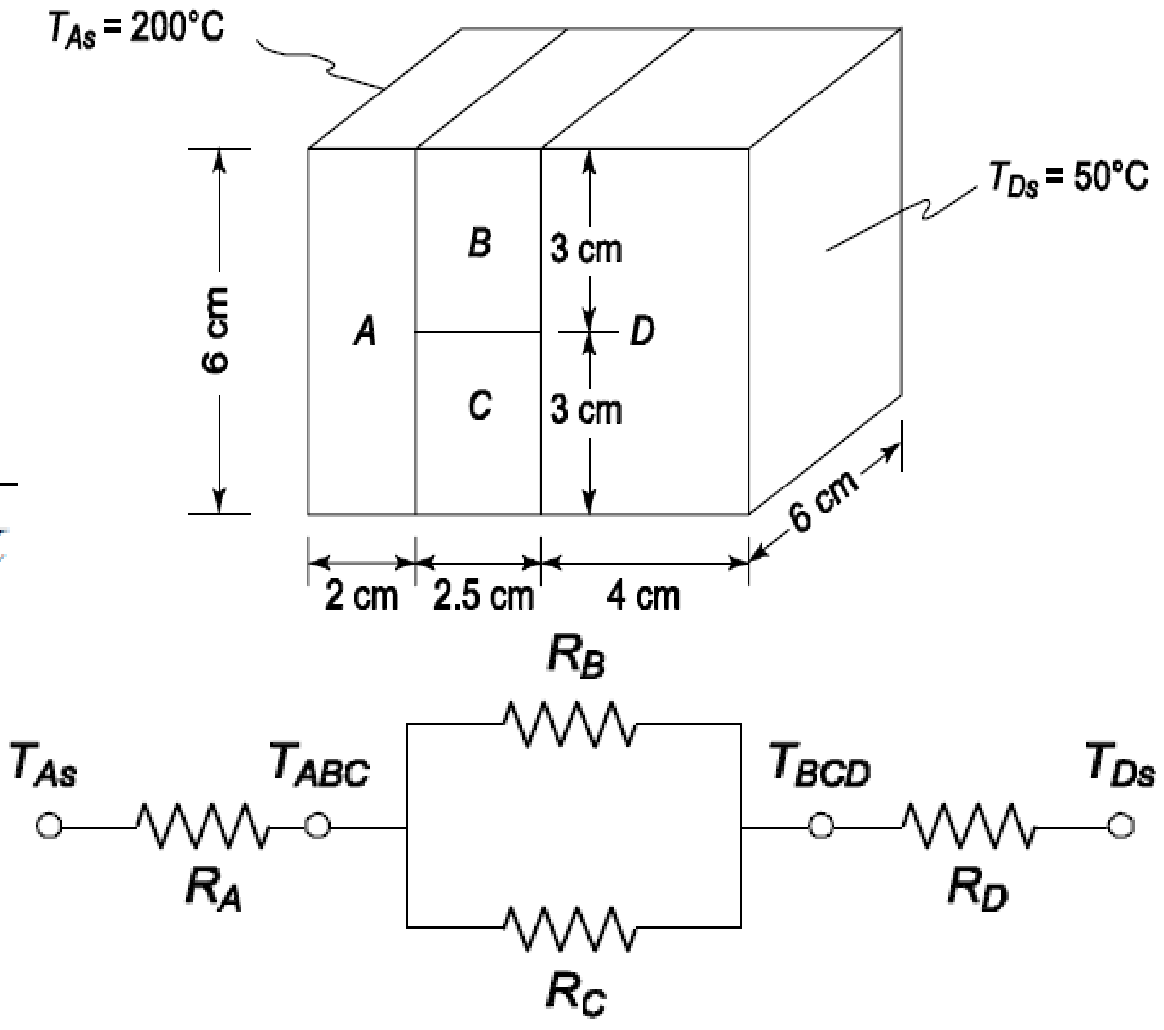
Problem 3

- A section of a composite wall
- Thermal conductivities
- k_A = 70 W/m.K
- k_B = 60 W/m.K
- k_C = 40 W/m.K
- k_D = 20 W/m.K
- Surface temperatures
- Left side (T_{As}) = 200 °C
- Right side (T_{Ds}) = 50 °C
- One dimensional conduction
- The system is in steady state
- The contact resistances between the materials is negligible



Problems

Problem 3



$$R_k = \frac{L}{Ak}$$

Problems

Problem 3

The total thermal resistance is

$$R_{\text{total}} = R_A + \frac{R_B\, R_C}{R_B + R_C} + R_D$$

$$R_{\text{total}} = 0.0794 + \frac{(0.2315)(0.3472)}{0.2315 + 0.3472} + 0.5556 \text{ K/W}$$

$$R_{\text{total}} = 0.7738 \text{ K/W}$$

The total rate of heat transfer through the composite wall is given by

$$q = \frac{\Delta T}{R_{\text{total}}} = \frac{200^{\circ}\text{C} - 50^{\circ}\text{C}}{0.7738\,\text{K/W}} = 194 \text{ W}$$

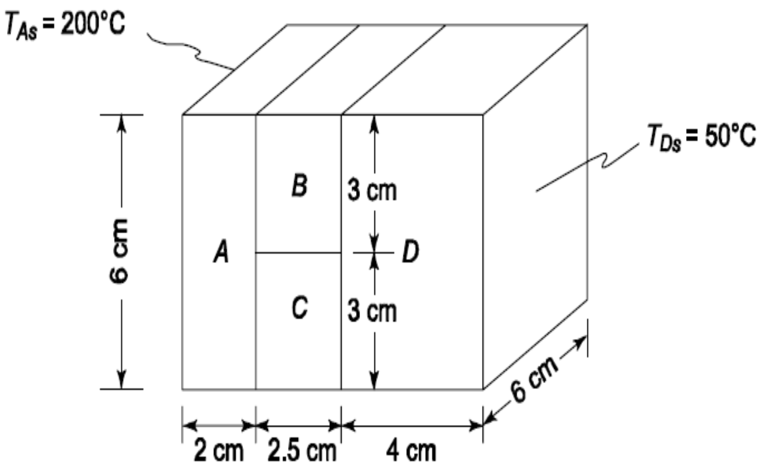
Problems

Problem 3

The average temperature at the interface between material *A* and materials *B* and *C* (T_{ABC}) can be determined by considering the conduction through material *A* alone.

$$q_{ka} = \frac{T_{As} - T_{ABC}}{R_A} = q$$

$$T_{ABC} = T_{As} - q\,R_A = 200°\text{C} - (194\ \text{W})\,(0.0794\ \text{K/W}) = 185°\text{C}$$



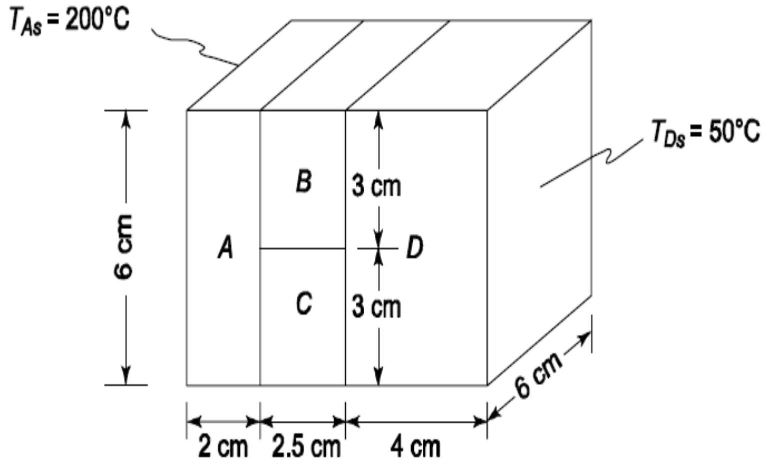
Problems

Problem 3

The average temperature at the interface between material *D* and materials *B* and *C* (T_{BCD}) can be determined by considering the conduction through material *D* alone

$$q_{kD} = \frac{T_{BCD} - T_{Ds}}{R_D} = q$$

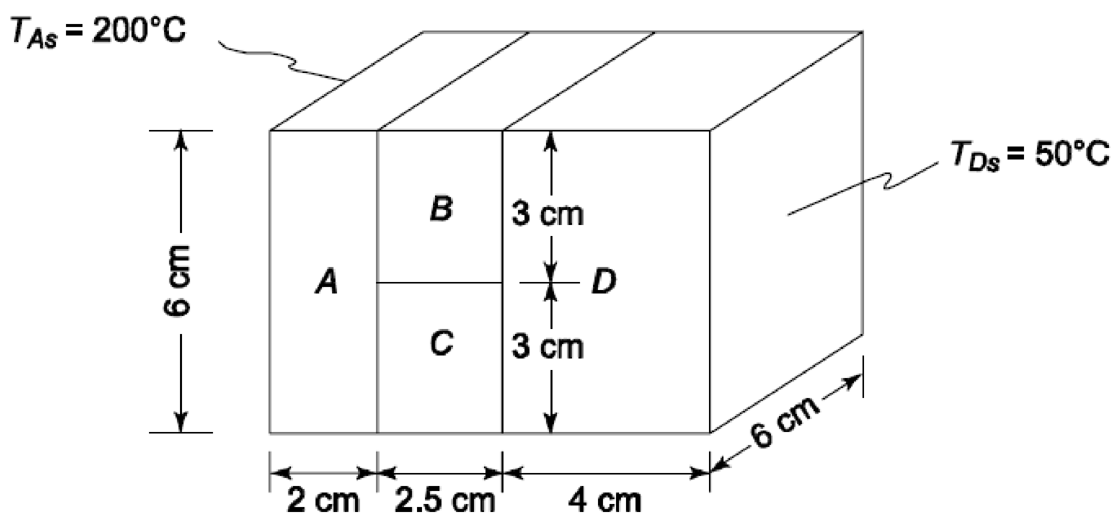
$$T_{BCD} = T_{Ds} + q\,R_D = 50°\text{C} + (194\ \text{W})\,(0.5556\ \text{K/W}) = 158°\text{C}$$



Problems

Problem 4

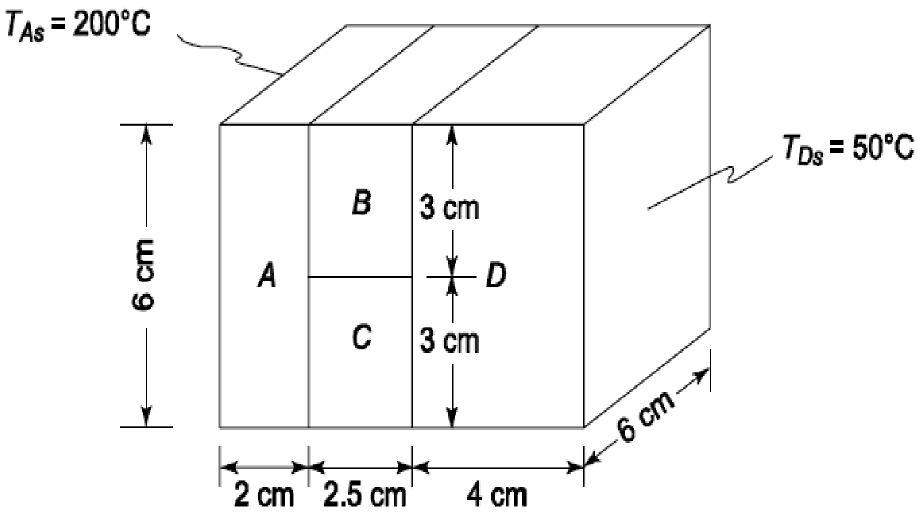
A composite wall (shown below) has uniform temperatures. If the thermal conductivities of the wall materials are: k_A = 70 W/m.K, k_B = 60 W/m.K, k_C = 40 W/m.K, and k_D = 20 W/m.K, ***and contact resistance at each interface R_i = 0.1 K/W***, determine the rate of heat transfer through this section of the wall and the temperatures at the interfaces. (Surfaces normal to heat transfer direction are isothermal).



Problems

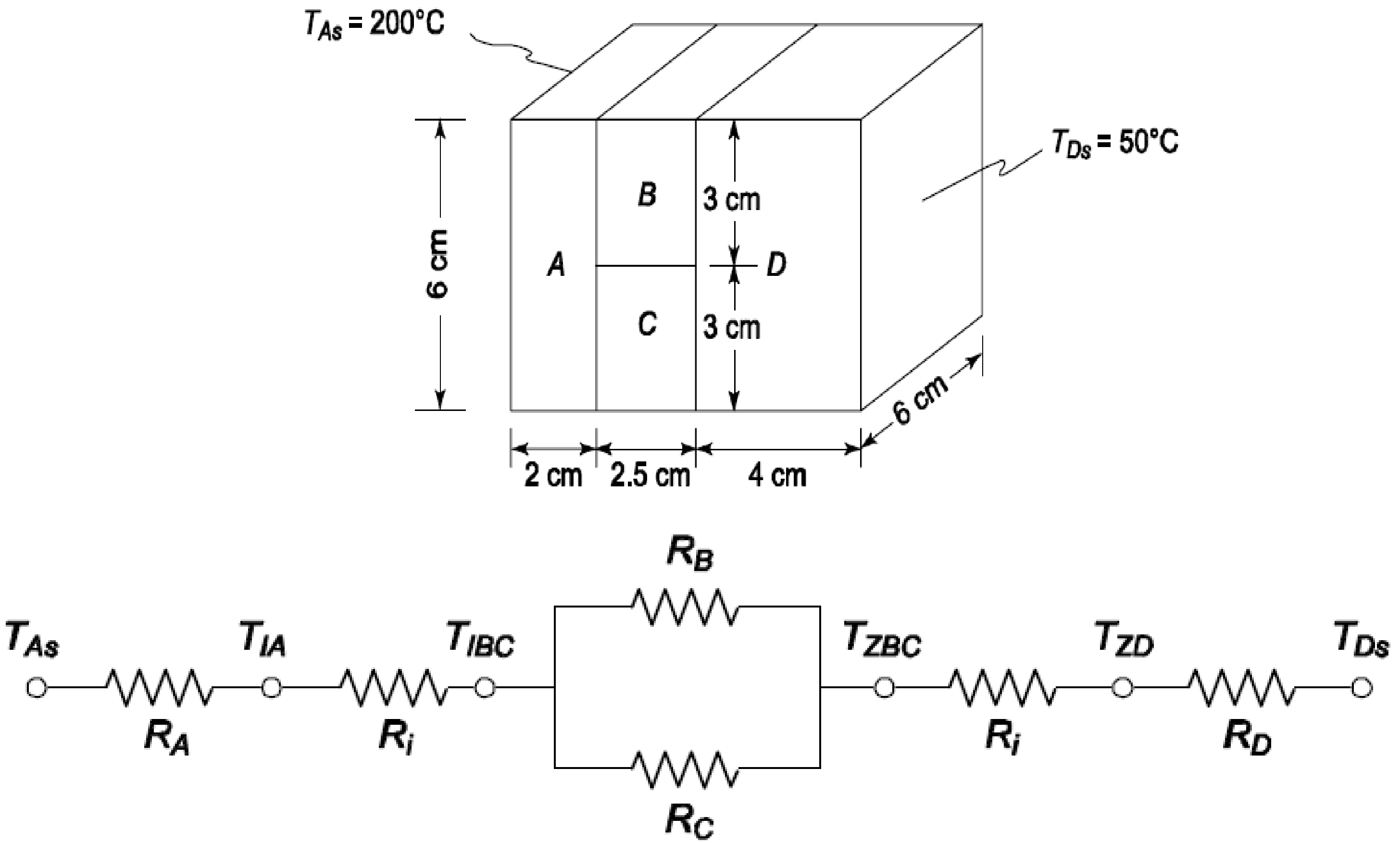
Problem 4

- A section of a composite wall
- Thermal conductivities
- k_A = 70 W/m.K
- k_B = 60 W/m.K
- k_C = 40 W/m.K
- k_D = 20 W/m.K
- Surface temperatures
- Left side (T_{As}) = 200 °C
- Right side (T_{Ds}) = 50 °C
- One dimensional conduction
- The system is in steady state
- ***Contact resistance at each interface (R_i) = 0.1 K/W***



Problems

Problem 4



Problems

Problem 4

(a) The total resistance for this system is

$$R_{\text{total}} = R_A + R_i + \frac{R_B R_C}{R_B + R_C} + R_i + R_D$$

$$R_{\text{total}} = 0.0794 + 0.1 + \frac{(0.2315)(0.3472)}{0.2315 + 0.3472} + 0.1 + 0.5556 \text{ K/W}$$

$$R_{\text{total}} = 0.9738 \text{ K/W}$$

The total rate of heat transfer through the composite wall is given by:

$$q = \frac{\Delta T}{R_{\text{total}}} = \frac{200°\text{C} - 50°\text{C}}{0.9738 \text{ K/W}} = 154 \text{ W}$$

Problems

Problem 4

(b) The average temperature on the *A* side of the interface between material *A* and material *B* and *C* (T_{1A}) can be determined by considering the conduction through material *A* alone.

$$q = \frac{T_{As} - T_{1A}}{R_A}$$

$$T_{1A} = T_{As} - q\, R_A = 200°\text{C} - (154 \text{ W})\,(0.0794 \text{ K/W}) = 188°\text{C}$$

The average temperature on the *B* and *C* side of the interface between material *A* and materials *B* and *C* (T_{1BC}) can be determined by considering the heat transfer through the contact resistance.

$$q = \frac{T_{1A} - T_{1BC}}{R_i}$$

$$T_{1BC} = T_{1A} - q\, R_i = 188°\text{C} - (154 \text{ W})\,(0.1 \text{ K/W}) = 172°\text{C}$$

Problems

Problem 4

The average temperature on the D side of the interface between material D and materials B and C (T_{2D}) can be determined by considering the conduction through material D alone.

$$q = \frac{T_{2D} - T_{Ds}}{R_D}$$

$$T_{2D} = T_{Ds} + q\,R_D = 50°\text{C} + (154\ \text{W})\,(0.5556\ \text{K/W}) = 136°\text{C}$$

The average temperature on the B and C side of the interface between material D and materials B and C (T_{2BC}) can be determined by considering the heat transfer through the contact resistance.

$$q = \frac{T_{2BC} - T_{2D}}{R_i}$$

$$T_{2BC} = T_{2D} + q\,R_i = 136°\text{C} + (154\ \text{W})\,(0.1\ \text{K/W}) = 151°\text{C}$$