CHAPTER 7
INTEREST RATES AND BOND VALUATION

Solutions to Questions and Problems

2. Price and yield move in opposite directions; if interest rates rise, the price of the bond will fall. This is because the fixed coupon payments determined by the fixed coupon rate are not as valuable when interest rates rise—hence, the price of the bond decreases.

3. The price of any bond is the PV of the interest payments, plus the PV of the par value. Notice this problem assumes an annual coupon. The price of the bond will be:
$P = €38(\{1 – [1/(1 + .047^{23})]\}/.047) + €1,000[1/(1 + .047)^{23}]$
$P = €875.09$

We would like to introduce shorthand notation here. Rather than write (or type, as the case may be) the entire equation for the PV of a lump sum, or the PVA equation, it is common to abbreviate the equations as:
$\text{PVIF}_{R,t} = 1/(1 + R)^t$

which stands for <u>P</u>resent <u>V</u>alue <u>I</u>nterest <u>F</u>actor

$\text{PVIFA}_{R,t} = (\{1 – [1/(1 + R)^t]\}/R)$

which stands for <u>P</u>resent <u>V</u>alue <u>I</u>nterest <u>F</u>actor of an <u>A</u>nnuity

These abbreviations are shorthand notation for the equations in which the interest rate and the number of periods are substituted into the equation and solved. We will use this shorthand notation in the remainder of the solutions key.

4. Here we need to find the YTM of a bond. The equation for the bond price is:

$P = ¥105,430 = ¥3,400(\text{PVIFA}_{R\%,16}) + ¥100,000(\text{PVIF}_{R\%,16})$

Notice the equation cannot be solved directly for *R*. Using a spreadsheet, a financial calculator, or trial and error, we find:
R = YTM = 2.97%

If you are using trial and error to find the YTM of the bond, you might be wondering how to pick an interest rate to start the process. First, we know the YTM has to be lower than the coupon rate since the bond is a premium bond. That still leaves a lot of interest rates to check. One way to get a starting point is to use the following equation, which will give you an approximation of the YTM:

Approximate YTM = [Annual interest payment + (Price difference from par/Years to maturity)]/
[(Price + Par value)/2]

Solving for this problem, we get:
Approximate YTM = [¥3,400 + (–¥5,430/16)]/[(¥105,430 + 100,000)/2] = .0298, or 2.98%

This is not the exact YTM, but it is close, and it will give you a place to start.

7. Here we are finding the YTM of a semiannual coupon bond. The bond price equation is:
$P = \$1,050 = \$26.50(\text{PVIFA}_{R\%,46}) + \$1,000(\text{PVIF}_{R\%,46})$

Since we cannot solve the equation directly for R, using a spreadsheet, a financial calculator, or trial and error, we find:
$R = 2.467\%$

Since the coupon payments are semiannual, this is the semiannual interest rate. The YTM is the APR of the bond, so:

YTM = 2 × 2.467%
YTM = 4.93%

8. Here we need to find the coupon rate of the bond. All we need to do is to set up the bond pricing equation and solve for the coupon payment as follows:
$P = \$1,045 = C(\text{PVIFA}_{2.65\%,29}) + \$1,000(\text{PVIF}_{2.65\%,29})$

Solving for the coupon payment, we get:
$C = \$28.74$

Since this is the semiannual payment, the annual coupon payment is:
2 × $28.74 = $57.49

And the coupon rate is the annual coupon payment divided by par value, so:

Coupon rate = $57.49/$1,000
Coupon rate = .0575, or 5.75%

14. The Fisher equation, which shows the exact relationship between nominal interest rates, real interest rates, and inflation is:
$(1 + R) = (1 + r)(1 + h)$

$h = [(1 + .123)/(1 + .08)] – 1$
$h = .0398$, or 3.98%

19. Any bond that sells at par has a YTM equal to the coupon rate. Both bonds sell at par, so the initial YTM on both bonds is the coupon rate, 7.3 percent. If the YTM suddenly rises to 9.3 percent:

$P_{\text{Sam}} = \$36.50(\text{PVIFA}_{4.65\%,6}) + \$1,000(\text{PVIF}_{4.65\%,6}) = \$948.67$

$P_{\text{Dave}} = \$36.50(\text{PVIFA}_{4.65\%,40}) + \$1,000(\text{PVIF}_{4.65\%,40}) = \$819.86$

The percentage change in price is calculated as:

Percentage change in price = (New price – Original price)/Original price

$\Delta P_{\text{Sam}}\% = (\$948.67 – 1,000)/\$1,000 = –.0513$, or –5.13%

$\Delta P_{\text{Dave}}\% = (\$819.86 – 1,000)/\$1,000 = –.1801$, or –18.01%

If the YTM suddenly falls to 5.3 percent:

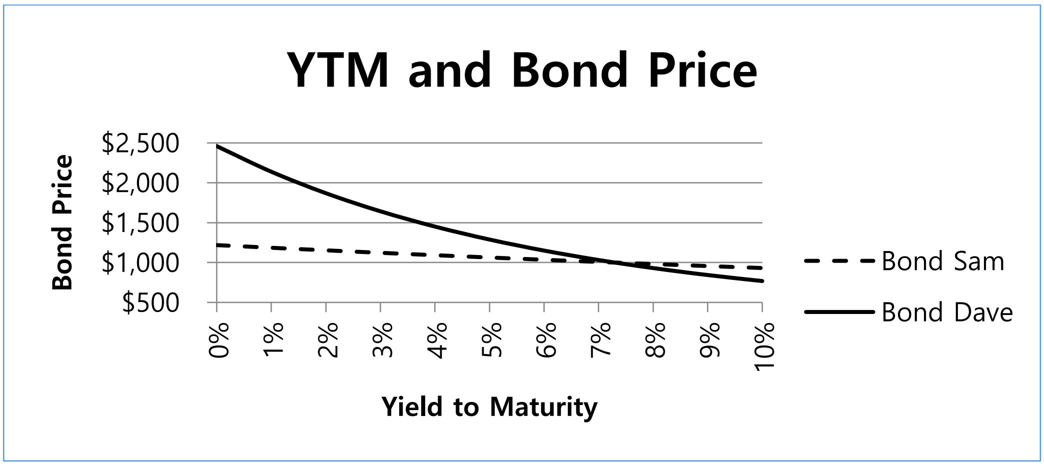
$P_{\text{Sam}} = \$36.50(\text{PVIFA}_{2.65\%,6}) + \$1,000(\text{PVIF}_{2.65\%,6}) = \$1,054.81$

P_{Dave} = \$36.50($PVIFA_{2.65\%,40}$) + \$1,000($PVIF_{2.65\%,40}$) = \$1,244.80

$\Delta P_{Sam}\%$ = (\$1,054.81 – 1,000)/\$1,000 = .0548, or 5.48%

$\Delta P_{Dave}\%$ = (\$1,244.80 – 1,000)/\$1,000 = .2448, or 24.48%

All else the same, the longer the maturity of a bond, the greater is its price sensitivity to changes in interest rates.



21. The current yield is:

Current yield = Annual coupon payment/Price
Current yield = \$64/\$943.10
Current yield = .0679, or 6.79%

The bond price equation for this bond is:
P_0 = \$943.10 = \$32($PVIFA_{R\%,36}$) + \$1,000($PVIF_{R\%,36}$)

Using a spreadsheet, financial calculator, or trial and error we find:

R = 3.480%

This is the semiannual interest rate, so the YTM is:

YTM = 2 × 3.480%
YTM = 6.96%

The effective annual yield is the same as the EAR, so using the EAR equation from the previous chapter:

Effective annual yield = $(1 + .03480)^2 - 1$
Effective annual yield = .0708, or 7.08%