EM PROBLEM SET 7

SOLUTIONS

1 (a) $\frac{\partial}{\partial x} \vec{B}(\vec{r}, t)$

$= \frac{\partial}{\partial x} \left(\vec{B}_0 e^{i(\vec{k}.\vec{r} - \omega t)} \right)$

$= \vec{B}_0 \frac{\partial}{\partial x} e^{i(k_x x + k_y y + k_z z - \omega t)}$

$= i k_x \vec{B}_0 e^{i(\vec{k}.\vec{r} - \omega t)}$

(b) Generalizing part (a) to $\frac{\partial}{\partial y}$ and $\frac{\partial}{\partial z}$,

$\frac{\partial}{\partial y} \vec{B}(\vec{r}, t) = i k_y \vec{B}_0 e^{i(\vec{k}.\vec{r} - \omega t)}$

$\frac{\partial}{\partial z} \vec{B}(\vec{r}, t) = i k_z \vec{B}_0 e^{i(\vec{k}.\vec{r} - \omega t)}$

Then $\vec{\nabla} \times \vec{B}(\vec{r}, t)$

$= \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) \times \vec{B}(\vec{r}, t)$

$= (i k_x, i k_y, i k_z) \times \vec{B}_0 e^{i(\vec{k}.\vec{r} - \omega t)}$

$$= i\vec{k} \times \vec{B}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

(c) $$\vec{E}(\vec{r},t) = \vec{E}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$\frac{\partial \vec{E}(\vec{r},t)}{\partial t} = -i\omega \vec{E}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

Substituting this result and the result of (b) into (*),

$$i\vec{k} \times \vec{B}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)} = -\frac{i\omega}{c^2}\vec{E}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

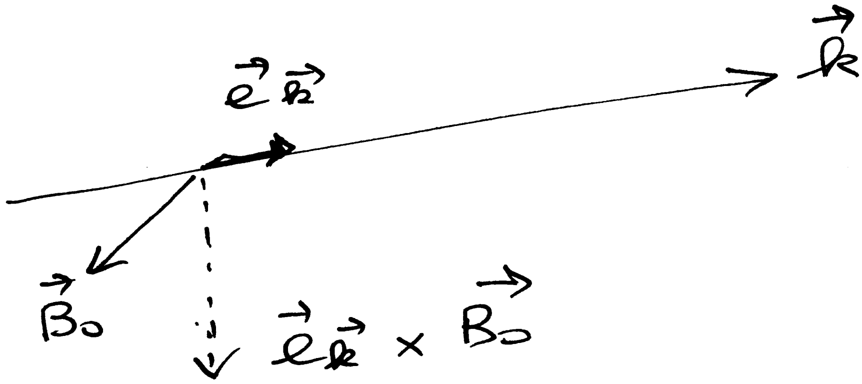
$$\Rightarrow \vec{k} \times \vec{B}_0 = -\frac{\omega}{c^2}\vec{E}_0$$

(d) Divide the result in (c) by $|\vec{k}|$,

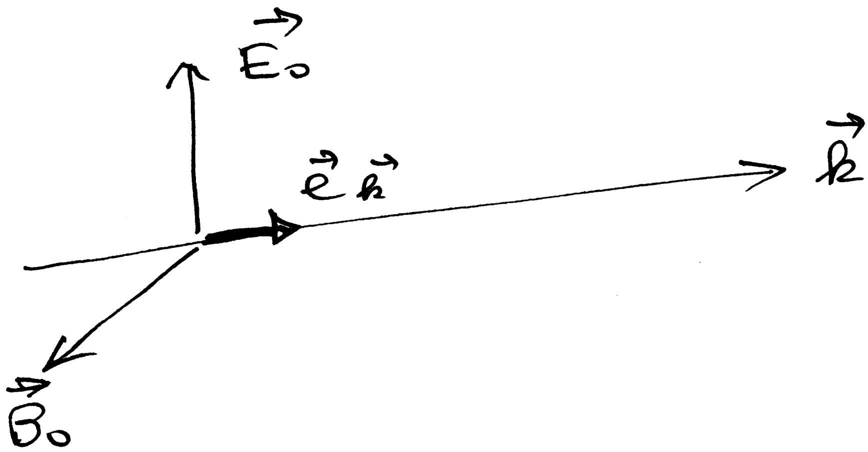
$$\underbrace{\frac{\vec{k}}{|\vec{k}|}}_{\vec{e}_{\vec{k}}} \times \vec{B}_0 = -\frac{1}{c^2}\underbrace{\frac{\omega}{|\vec{k}|}}_{c}\vec{E}_0$$

$$\Rightarrow \vec{e}_{\vec{k}} \times \vec{B}_0 = -\frac{1}{c}\vec{E}_0$$

(e) $\vec{E}_0 = -c\, \vec{e}_{\vec{k}} \times \vec{B}_0$



$-\vec{e}_k \times \vec{B}_0$ points in the opposite direction to $\vec{e}_k \times \vec{B}_0$



(f). Since $\vec{e}_k \perp \vec{B}_0$

$$|\vec{e}_{\vec{k}} \times \vec{B}_0| = |\vec{e}_{\vec{k}}|\,|\vec{B}_0|$$
$$= |\vec{B}_0|$$

since $\vec{e}_{\vec{k}}$ is a unit vector

So $\vec{E}_0 = -c\, \vec{e}_{\vec{k}} \times \vec{B}_0$

$\Rightarrow |\vec{E}_0| = c\,|\vec{B}_0|$