PROBLEM SET 8

SOLUTIONS

(a) Flux through loop

$$\Phi = \int_S \vec{B}(\vec{r}) \cdot d\vec{S}$$

Take S to be the square whose boundary is the loop, its area is A. Since $\vec{B}$ perpendicular to the plane of the loop, $\vec{B} \cdot d\vec{S} = B\, dS$

$$\Rightarrow \Phi = B \underbrace{\int dS}_{\text{area A of loop}}$$

$$= B \cdot A$$

(b) Now, the angle between $\vec{B}$ and $d\vec{S}$ (the latter is still perpendicular to the loop) is θ

$$\Rightarrow \vec{B} \cdot d\vec{S} = B\, dS \cos\theta$$

$\Rightarrow$ magnetic flux

$$\Phi = \int \vec{B} \cdot d\vec{s}$$

$$= B \cos\theta \int ds$$

$$= B A \cos\theta$$

[check : when $\theta = 0$, $\Phi = BA$]

(c) $\Phi(t) = B A \cos\theta(t)$

$$\Rightarrow |\mathcal{E}| = \left| \frac{d\Phi(t)}{dt} \right|$$

$$= BA \left| -\sin\theta(t) \cdot \frac{d\theta(t)}{dt} \right|$$

$$= BA\omega \sin\omega t$$