EM CLASS TEST 2

VERSION 1 – SOLUTIONS

1. A straight wire of radius R carries a current I parallel to the wire which is uniformly distributed across the wire. The magnetic field produced by the current curls around the direction of the wire, and the magnitude of the magnetic field depends only on the distance r from the centre of the wire (both inside and outside the wire). The magnitude $B(r)$ of the magnetic field for $r > R$ (i.e **outside the wire**) is:

   A. $B(r) = \frac{I}{\pi\epsilon_0 c^2 r^2}$
   B. $B(r) = \frac{I\,r}{2\pi\epsilon_0 c^2 R^2}$
   C. $B(r) = \frac{I}{2\pi\epsilon_0 c^2\, r}$
   D. $B(r) = \frac{I}{2\pi\epsilon_0 c^2 R}$

2. A straight wire of radius R carries a current I parallel to the wire which is uniformly distributed across the wire. The magnetic field produced by the current curls around the direction of the wire, and the magnitude of the magnetic field depends only on the distance r from the centre of the wire (both inside and outside the wire). The magnitude $B(r)$ of the magnetic field for $r < R$ (i.e **inside the wire**) is:

   A. $B(r) = \frac{I}{\pi\epsilon_0 c^2 r^2}$
   B. $B(r) = \frac{I\,r}{2\pi\epsilon_0 c^2 R^2}$
   C. $B(r) = \frac{I}{2\pi\epsilon_0 c^2\, r}$
   D. $B(r) = \frac{I}{2\pi\epsilon_0 c^2 R}$

3. A circular loop of wire with radius R is in a magnetic field which is perpendicular to the plane of the circle. The magnitude of the magnetic field is time dependent, with time dependence $B(t) = B_0\, t$, where B_0 is constant. The magnitude of the induced EMF in the circular loop of wire is:

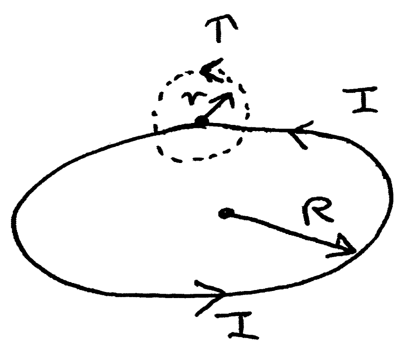
   A. $\pi R^2 B_0$
   B. B_0
   C. $2\pi R B_0$
   D. $\pi R^2 B_0\, t$

4. Using the Lorentz force law, the force on a particle with charge q moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$ is $\vec{F} = q\,\vec{v}\times\vec{B}$. Which of the following statements is CORRECT:

   A. The work done by the magnetic field on the particle in time dt is $q\,\vec{v}\times\vec{B}\,dt$
   B. The work done by the magnetic field on the particle in time dt is $q\,|\vec{v}|\,|\vec{B}|\,dt$
   C. The magnetic field does no work on the particle because the force is parallel to the displacement $\vec{v}\,dt$ that occurs in time dt.
   D. The magnetic field does no work on the particle because the force is perpendicular to the displacement $\vec{v}\,dt$ that occurs in time dt.

5. Consider a wire loop lying in the plane of this page. A magnetic field pointing out of the page is decreasing in magnitude. Which of the following statements is correct:
   A. The magnetic flux through the loop is increasing and Lenz's law implies the induced current is clockwise.
   B. The magnetic flux through the loop is increasing and Lenz's law implies the induced current is counter clockwise.
   C. The magnetic flux through the loop is decreasing and Lenz's law implies the induced current is clockwise.
   D. The magnetic flux through the loop is decreasing and Lenz's law implies the induced current is counter clockwise.

6. A circular current loop of radius R carrying a current I. Consider an imaginary circular path Γ of radius r centred on a point in the current loop, with the plane in which Γ lies perpendicular to the direction of the current, as shown in the diagram below:



The circulation of the magnetic field around Γ, namely $\oint_\Gamma \vec{B}(\vec{r}) \cdot d\vec{\ell}$, is:
   A. $\frac{I}{2\pi\epsilon_0 c^2 r}$
   B. $\frac{I}{\pi\epsilon_0 c^2 r^2}$
   C. $\frac{I}{\epsilon_0 c^2 r}$
   D. $\frac{I}{\epsilon_0 c^2}$

7. The Maxwell equation $\vec{\nabla} \cdot \vec{B}(\vec{r}) = 0$ implies:
   A. The circulation of the magnetic field vanishes.
   B. The circulation of the magnetic field is non vanishing
   C. There are no point "sources" or "sinks" of magnetic fields (isolated magnetic charges or magnetic monopoles) as the field lines are not diverging or converging.
   D. There are no point "sources" or "sinks" of magnetic fields (isolated magnetic charges or magnetic monopoles) as the magnetic field has no circulation.

8. Consider the one-dimensional wave equation

$$\frac{\partial^2 f(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 f(x,t)}{\partial t^2}.$$

The function $f(x,t) = A\, e^{i(kx-\omega t)}$ satisfies this equation provided:

A. $\omega\, k = v$

B. $\frac{k}{\omega} = v$

C. $\frac{\omega}{k} = v$

D. $\frac{1}{\omega k} = v$

9. For the magnetic field $\vec{B}(\vec{r},t) = \vec{B}_0\, e^{i(\vec{k}\cdot\vec{r}-\omega t)}$, the Maxwell equation $\nabla \cdot \vec{B}(\vec{r},t) = 0$ implies:

A. $\vec{k}\,|\vec{B}_0| = 0$

B. $\vec{k} \times \vec{B}_0 = 0$

C. $|\vec{k}|\,\vec{B}_0 = 0$

D. $\vec{k} \cdot \vec{B}_0 = 0$

10. Consider a closed surface S enclosing a volume V. The net current flowing through the surface S is $\oint_S \vec{j}(\vec{r},t) \cdot d\vec{S}$, were $\vec{j}(\vec{r},t)$ is the current density. A positive value for the surface integral means a net outflow of charge, and a negative value means a net inflow of charge. The total charge enclosed by the surface is $\int_V \rho(\vec{r},t)\, d^3\vec{r}$, where $\rho(\vec{r},t)$ is the charge density. Conservation of charge requires that $\oint_S \vec{j}(\vec{r},t) \cdot d\vec{S}$ is equal to:

A. $-\frac{d}{dt}\int_V \rho(\vec{r},t)\, d^3\vec{r}$

B. $\frac{d}{dt}\int_V \vec{\nabla}\rho(\vec{r},t)\, d^3\vec{r}$

C. $\frac{d}{dt}\int_V \rho(\vec{r},t)\, d^3\vec{r}$

D. $\int_V \rho(\vec{r},t)\, d^3\vec{r}$