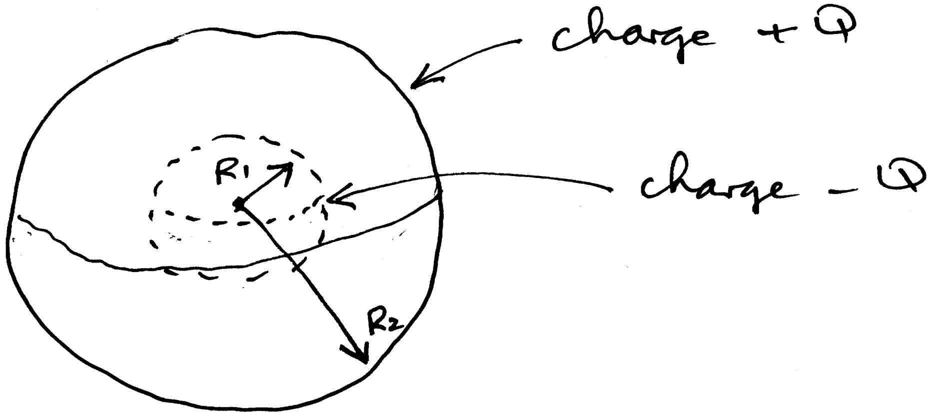
EM PROBLEM SET 9 – 2019

1. A spherical capacitor consists of two concentric spherical conducting shells with charges $+Q$ and $-Q$ on the shells:



(a) Determine the charge density (charge per unit surface area) on the inner sphere and on the outer sphere.

(b) The symmetry of the charge distribution means any electric field must be radial (inwards or outwards pointing) and have a magnitude distance r from the centre.

By choosing an appropriate Gaussian surface, use Gauss's law to determine the magnitude $E(r)$ and direction of the electric field for

(i) $r < R_1$

(ii) $R_1 < r < R_2$

(iii) $r > R_2$

(c) Determine the work required to move a point charge q from the inner conductor to the outer conductor.

(d) Hence determine the electric potential difference between the conductors, and the capacitance C via the relation $\Delta V = \frac{1}{C} Q$

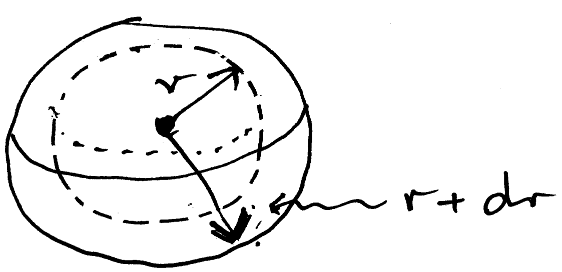
(e) The energy density (energy per unit volume) in an electric field is

$$u(\vec{r}) = \frac{\varepsilon_0}{2} |\vec{E}(\vec{r})|^2.$$

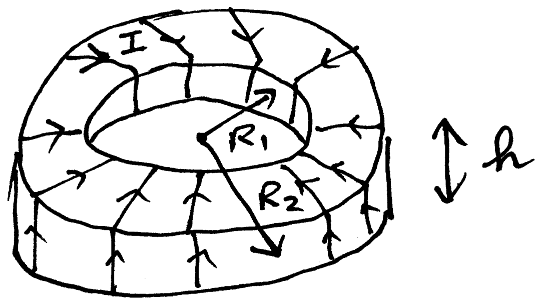
Show that the energy stored in the electric field in the spherical capacitor is

$$U = \frac{1}{2} \frac{Q^2}{C}$$

[Hint: consider an infinitesimal spherical shell between r and $r+dr$, which has volume $4\pi r^2\,dr$, and work out the energy contained in this infinitesimal shell; then integrate.].



2. Consider a rectangular torus of inner radius R_1, outer radius R_2, and height h, wound with N turns of wire carrying a current I.



(a) The symmetry of the current distribution means that the magnetic field inside the torus is parallel to the axis of the torus and depends in magnitude only on the distance r from the centre of the torus. By choosing an appropriate Amperian loop, determine $B(r)$ for

(i) $r < R_1$

(ii) $R_1 < r < R_2$

(iii) $r > R_2$

(b) Compute the magnetic flux Φ through the surface inside of one turn of wire.

<u>Hint</u>: first determine the magnetic flux through a vertical slice between r and $r+dr$, then integrate

(c) The magnetic flux through the torus is N times the flux through one turn of wire.

Use the definition $\Phi = L I$ to determine the inductance of the toroidal solenoid.

(d) Determine the energy contained in the magnetic field using the fact that the energy density is

$$u(\vec{r}) = \frac{1}{2} \varepsilon_0 c^2 |\vec{B}(\vec{r})|^2$$

<u>Hint</u>: first determine the energy in a vertical slice between r and

r + dr, then integrate.

(d) Show that the total energy stored in the magnetic field can be expressed in the form

$$U = \frac{1}{2} L I^2$$