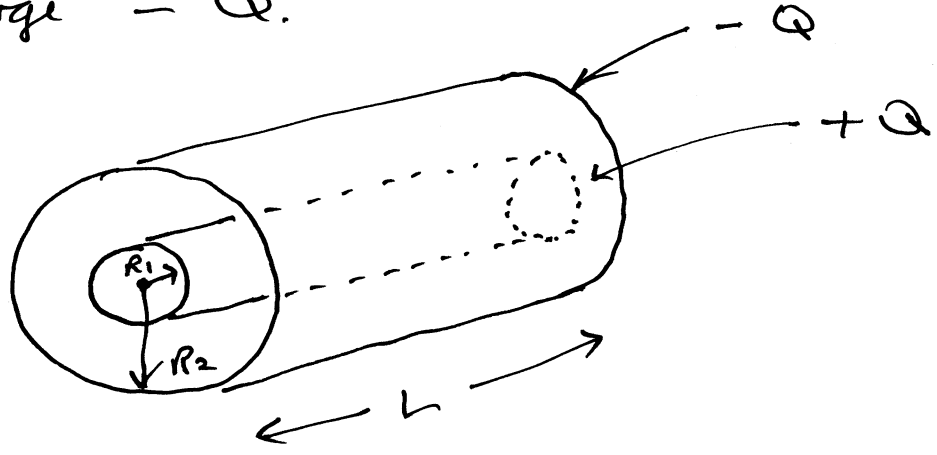
PROBLEM SET 10 — 2019

1. Consider a concentric pair of cylindrical conductors of radius R_1 and R_2 and length L. Suppose the inner conductor has a charge $+Q$ and the outer conductor has charge $-Q$.



Due to the symmetry of the charge distribution, any electric field will be radial (inwards or out-wards). Due to the symmetry of the charge distribution, any electric field is radial (pointing inwards or outwards) and has a

magnitude $E(r)$ that depends only on the radial distance r from the axis of the cylinders.

(a) By choosing an appropriate Gaussian surface and using Gauss's law, compute $E(r)$ and the direction of the electric field for

(i) $r < R_1$

(ii) $R_1 < r < R_2$

(iii) $r > R_2$

(b) Compute the potential difference ΔV between the inner and outer cylinders, and indicate which cylinder has the higher electric potential

(Note $V(\vec{r}_2) - V(\vec{r}_1) = -\int_T \vec{E}(\vec{r}) \cdot d\vec{l}$

where T is a path from $\vec{r}_1$ to $\vec{r}_2$)

(c) What is the amount of work required to move a charge $+q$ from

(d) Express $E(r)$ in terms of ΔV rather than $\frac{Q}{L}$.

(e) Compute the capacitance C of the pair of cylinders using

$$\Delta V = \frac{1}{C} Q$$

(f) Determine the electric potential energy stored in the electric field

Note: energy density $u(r) = \frac{\varepsilon_0}{2} E(r)^2$.

First compute the energy in in thin cylindrical shell between r and $r + dr$, then integrate from $r = R_1$ to $r = R_2$