HEAT TRANSFER

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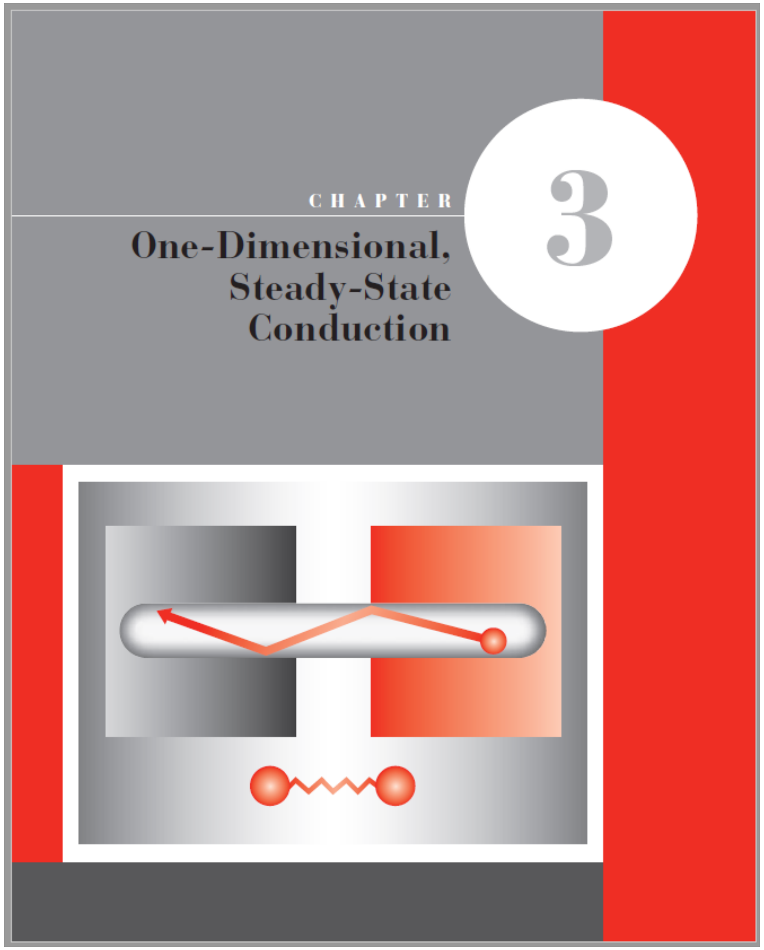
Heat Transfer from Extended Surfaces

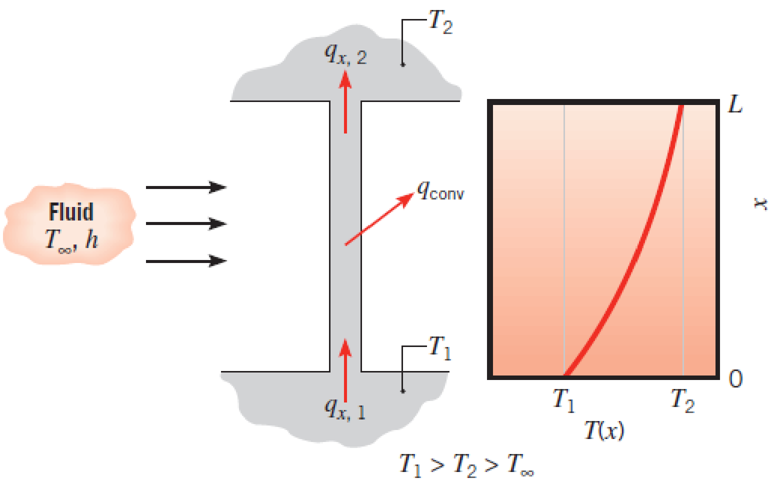


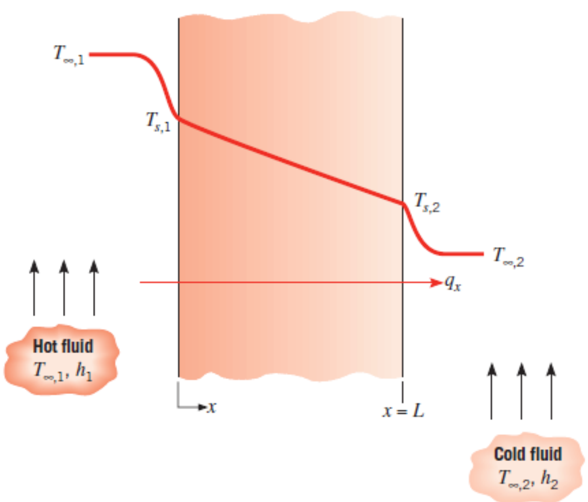
Heat Transfer from Extended Surfaces

1. Introduction

The term ***extended surface*** is usually used for figure out ***a special case of heat transfer***; heat transfer by conduction within a solid and heat transfer by convection (and/or radiation) from the boundaries of the solid.

- As compared to the concepts studied before, for an extended surface, ***the direction of heat transfer from the boundaries is perpendicular to the principal direction of heat transfer in the solid.***
- Until now, we have considered heat transfer from the boundaries of a solid to be in the ***same direction as heat transfer by conduction in the solid***.

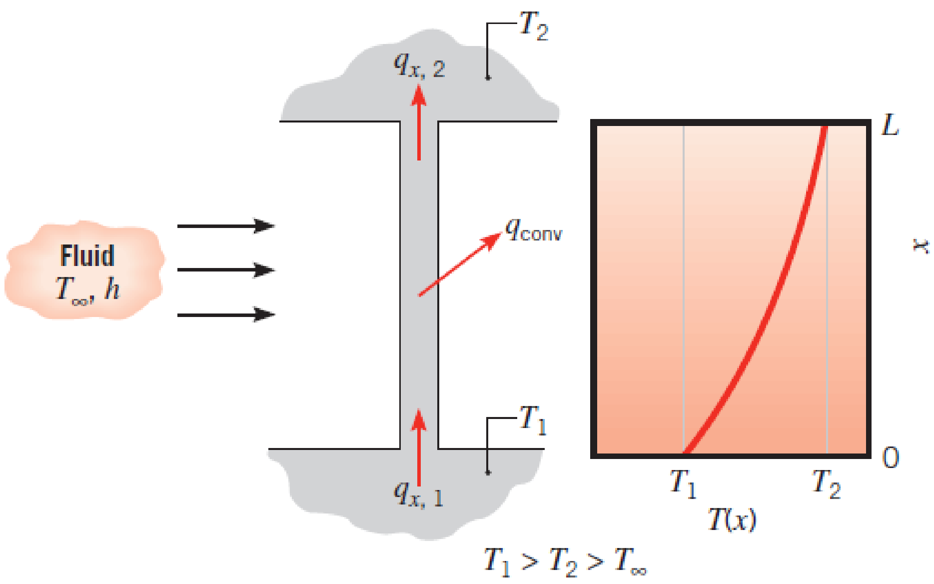




Heat Transfer from Extended Surfaces

1. Introduction

Consider a strut that connects two walls at different temperatures and across which there is fluid flow.



- ***With $T_1 > T_2$, temperature gradients in the x-direction sustain heat transfer by conduction.***
- ***with $T_1 > T_2 > T_\infty$, there will be concurrent transfer to the fluid by convection.***
- ***The magnitude of temperature gradient, accordingly, will decease by increasing x.***
- ***This is very important in several applications, like fins (Fin-enhanced heat transfer).***

Heat Transfer from Extended Surfaces

1. Introduction

Fin-enhanced heat transfer

Two ways of increasing heat transfer rate in Figure 3.13a:

- Increasing the fluid velocity to increase the convection coefficient h.
- The fluid temperature T_∞ could be reduced.
- ***Increasing the surface area across which the convection occurs by employing fins (Fig. 3.13b).***

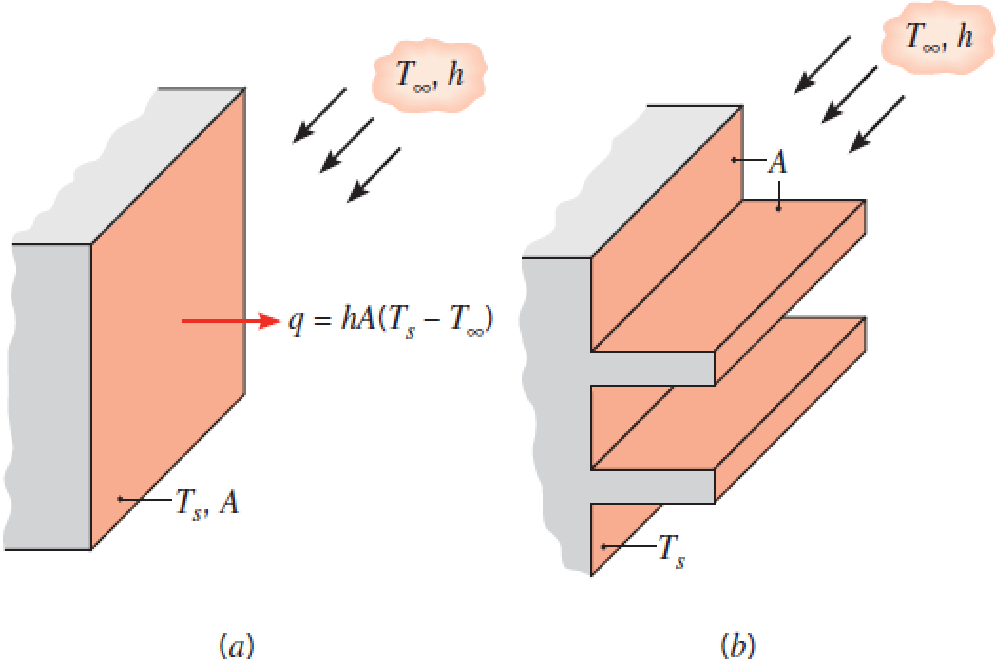


FIGURE 3.13 Use of fins to enhance heat transfer from a plane wall. (*a*) Bare surface. (*b*) Finned surface.

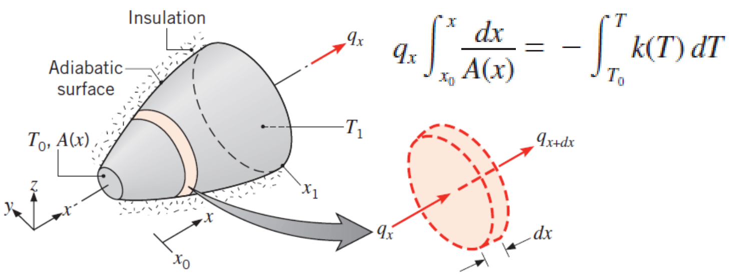
Heat Transfer from Extended Surfaces

2. A General Conduction Analysis

It is first important to know to which extent extended surfaces or fin arrangements could improve heat transfer from a surface to the surrounding fluid. In order to find that, temperature distribution along the fin should be determined by considering some assumptions:

1. ***One-dimensional conduction conditions in x-direction.***
2. ***The rate of conduction at any point is equal to the rate to convection at that point.***
3. ***Temperature is uniform throughout the thickness of the fin (it is only a function to x).***
4. ***Steady-state conditions.***
5. ***Thermal conductivity is constant.***
6. ***The radiation from the surface is negligible.***
7. ***No heat generation.***
8. ***Convection heat transfer coefficient (h) is uniform over the surface.***

Heat Transfer from Extended Surfaces



2. A General Conduction Analysis

By applying conservation of energy:

$$q_x = q_{x+dx} + dq_{\text{conv}}$$

From Fourier's law we know that:

$$q_x = -kA_c \frac{dT}{dx}$$

where A_c is the *cross-sectional* area, which may vary with x. Since the conduction heat rate at $x + dx$ may be expressed as:

$$q_{x+dx} = q_x + \frac{dq_x}{dx}\, dx$$

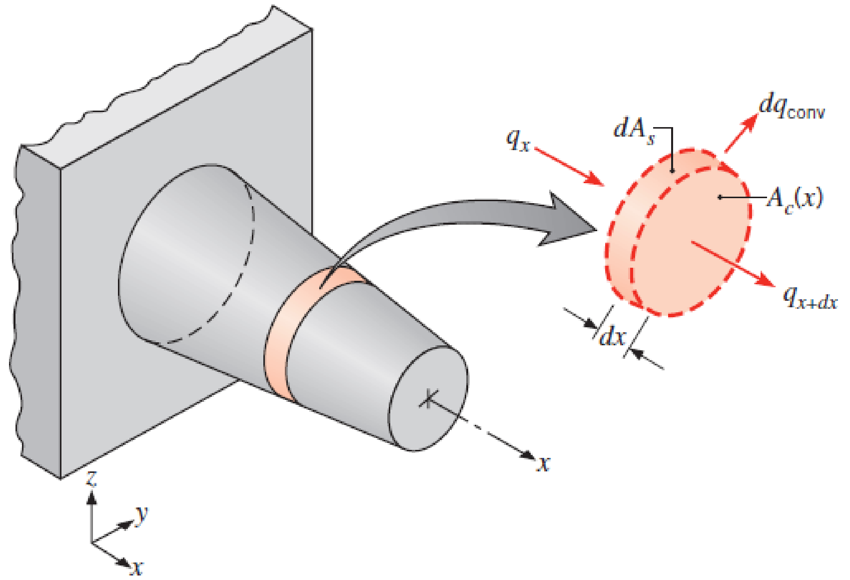


FIGURE 3.16 Energy balance for an extended surface.

Heat Transfer from Extended Surfaces

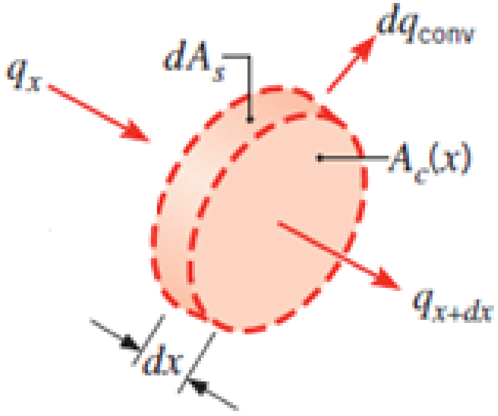
2. A General Conduction Analysis

$$q_{x+dx} = -kA_c \frac{dT}{dx} - k \frac{d}{dx}\left(A_c \frac{dT}{dx}\right) dx$$

For convection:

$$dq_{\text{conv}} = h dA_s (T - T_\infty)$$

where dA_s is the *surface* area of the differential element.



$$\frac{d}{dx}\left(A_c \frac{dT}{dx}\right) - \frac{h}{k}\frac{dA_s}{dx}(T - T_\infty) = 0$$

Heat Transfer from Extended Surfaces

2. A General Conduction Analysis

- This result provides a general form of the energy equation for an extended surface.
- ***Its solution for appropriate boundary conditions provides the temperature distribution.***

$$\boxed{\frac{d^2T}{dx^2} + \left(\frac{1}{A_c}\frac{dA_c}{dx}\right)\frac{dT}{dx} - \left(\frac{1}{A_c}\frac{h}{k}\frac{dA_s}{dx}\right)(T - T_\infty) = 0}$$

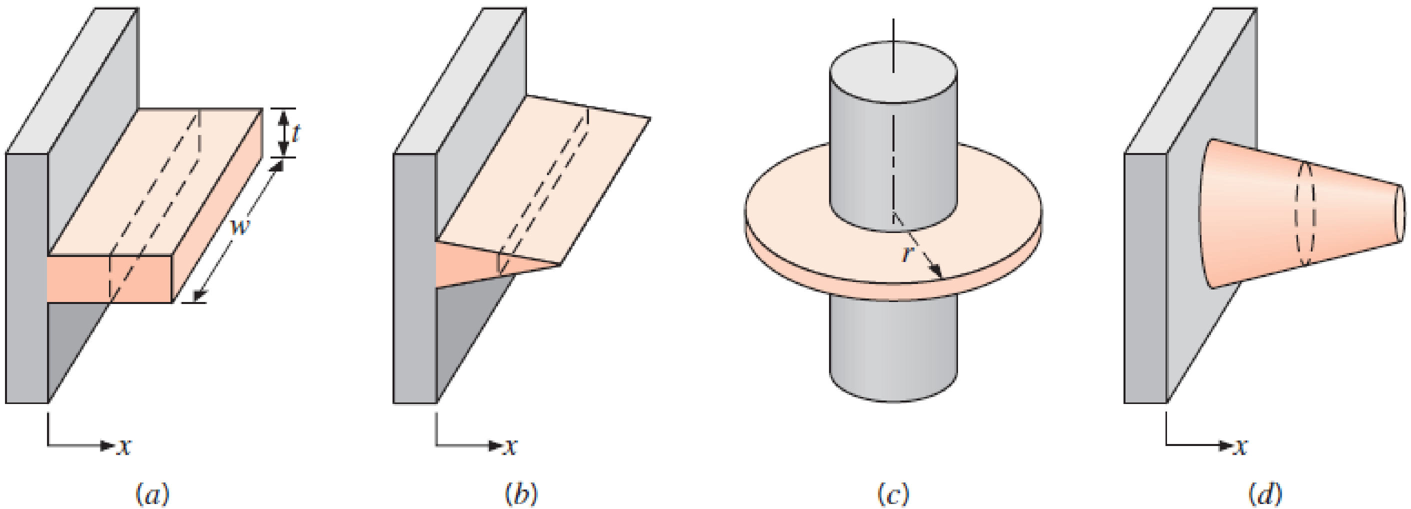


FIGURE 3.15 Fin configurations. (*a*) Straight fin of uniform cross section. (*b*) Straight fin of nonuniform cross section. (*c*) Annular fin. (*d*) Pin fin.

Heat Transfer from Extended Surfaces

2. 1. Fins of Uniform Cross-Sectional Area

The simplest case of straight rectangular and pin fins of uniform cross section.

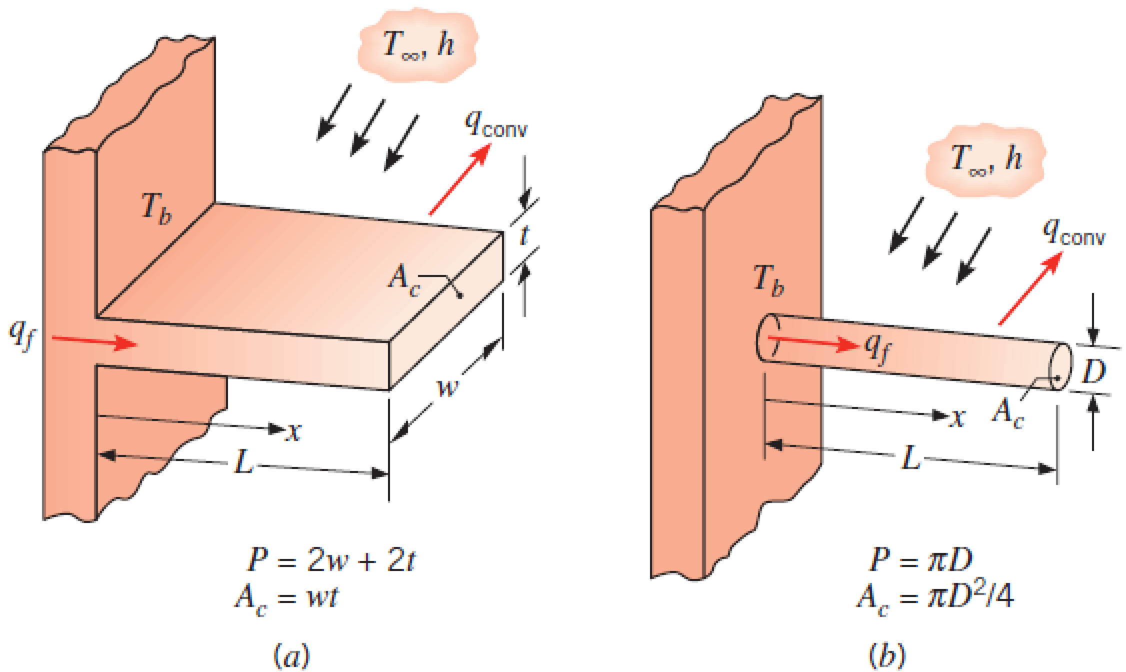
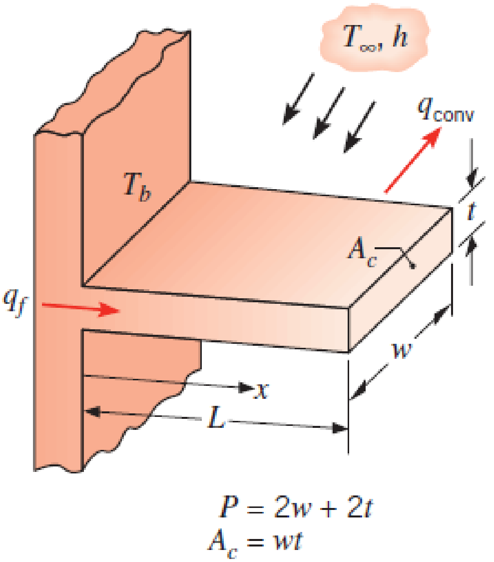


FIGURE 3.17 Straight fins of uniform cross section. (*a*) Rectangular fin. (*b*) Pin fin.

Heat Transfer from Extended Surfaces

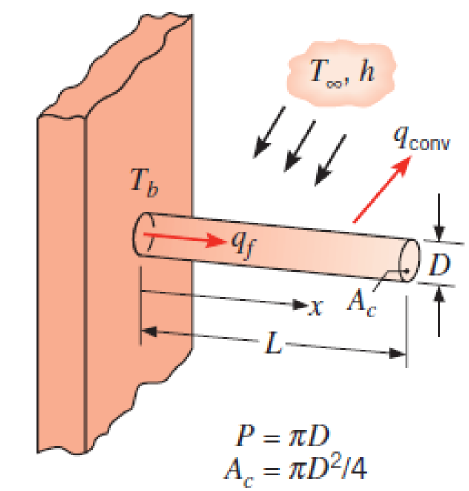
2. 1. Fins of Uniform Cross-Sectional Area



- Each fin is attached to a base surface of temperature $T(0) = T_b$ and extends into a fluid of temperature T_∞.
- A_c is a constant.
- A_s is equal to $P.x$, *where P is the perimeter.*

$$\boxed{\frac{d^2T}{dx^2} + \left(\frac{1}{A_c}\frac{dA_c}{dx}\right)\frac{dT}{dx} - \left(\frac{1}{A_c}\frac{h}{k}\frac{dA_s}{dx}\right)(T - T_\infty) = 0}$$

$$\frac{d^2T}{dx^2} - \frac{hP}{kA_c}(T - T_\infty) = 0$$



Heat Transfer from Extended Surfaces

2. 1. Fins of Uniform Cross-Sectional Area

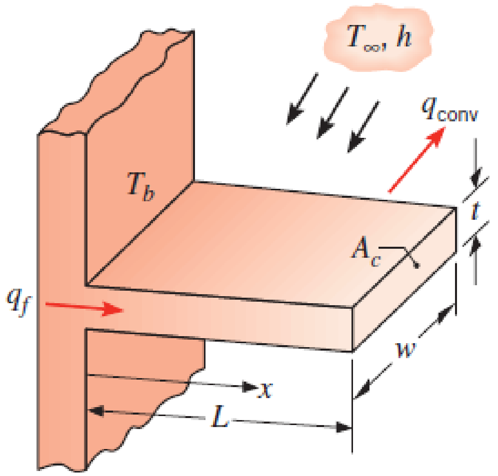
To simplify the form of this equation, we transform the dependent variable by defining an *excess temperature (θ)* as:

$$\theta(x) \equiv T(x) - T_\infty$$

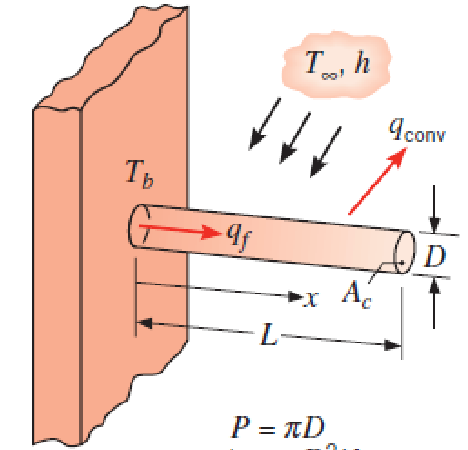
$$\frac{d^2\theta}{dx^2} - m^2\theta = 0 \qquad m^2 \equiv \frac{hP}{kA_c}$$

This is ordinary 2nd order differential equation, the general solution of this equation is:

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$$



$P = 2w + 2t$
$A_c = wt$



$P = \pi D$
$A_c = \pi D^2/4$

Heat Transfer from Extended Surfaces

2. 1. Fins of Uniform Cross-Sectional Area

Boundary conditions:

$$\boxed{\theta(x) = C_1 e^{mx} + C_2 e^{-mx}}$$

1. The temperature at the *base* of the fin ($x = 0$):

$$\theta(0) = T_b - T_\infty \equiv \theta_b \quad \Rightarrow \quad \boxed{\theta_b = C_1 + C_2}$$

2. At the tip of the fine ($x=L$): there are 4 cases:

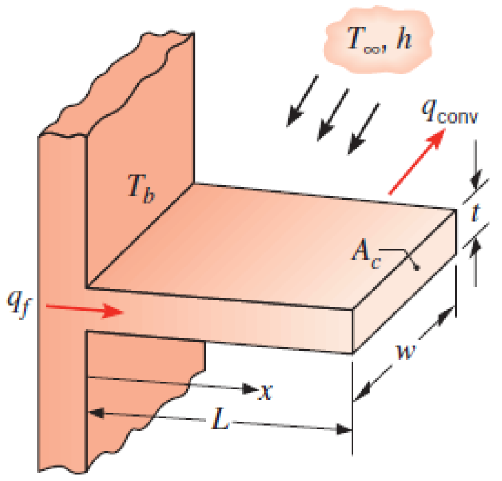
➢ Convection through A_c:

$$hA_c[T(L) - T_\infty] = -kA_c \left.\frac{dT}{dx}\right|_{x=L} \quad \Rightarrow \quad h\theta(L) = -k\left.\frac{d\theta}{dx}\right|_{x=L}$$

$$\boxed{h(C_1 e^{mL} + C_2 e^{-mL}) = km(C_2 e^{-mL} - C_1 e^{mL})}$$

$$\boxed{\frac{\theta}{\theta_b} = \frac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}}$$

$$m^2 \equiv \frac{hP}{kA_c}$$



$P = 2w + 2t$

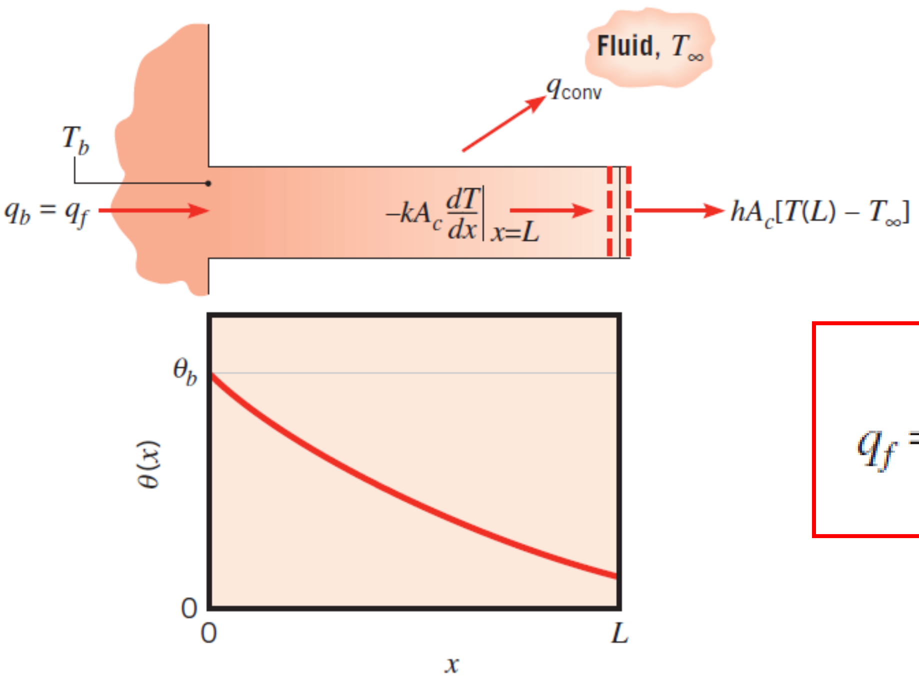
$A_c = wt$

Heat Transfer from Extended Surfaces

2. 1. Fins of Uniform Cross-Sectional Area

$$\boxed{\frac{\theta}{\theta_b} = \frac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}}$$

$$m^2 \equiv \frac{hP}{kA_c}$$



$$q_f = q_b = -kA_c \frac{dT}{dx}\Big|_{x=0} = -kA_c \frac{d\theta}{dx}\Big|_{x=0}$$

$$\boxed{q_f = \sqrt{hPkA_c}\,\theta_b \frac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}}$$

Heat Transfer from Extended Surfaces

2. 1. Fins of Uniform Cross-Sectional Area

$$\boxed{\theta(x) = C_1 e^{mx} + C_2 e^{-mx}} \quad + \quad \theta(0) = T_b - T_\infty \equiv \theta_b$$

TABLE 3.4 Temperature distribution and heat loss for fins of uniform cross section

Case	Tip Condition ($x = L$)	Temperature Distribution θ/θ_b	Fin Heat Transfer Rate q_f
A	Convection heat transfer: $h\theta(L) = -k\,d\theta/dx\vert_{x=L}$	$\dfrac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$ (3.75)	$M\dfrac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$ (3.77)
B	Adiabatic: $d\theta/dx\vert_{x=L} = 0$	$\dfrac{\cosh m(L-x)}{\cosh mL}$ (3.80)	$M \tanh mL$ (3.81)
C	Prescribed temperature: $\theta(L) = \theta_L$	$\dfrac{(\theta_L/\theta_b)\sinh mx + \sinh m(L-x)}{\sinh mL}$ (3.82)	$M\dfrac{(\cosh mL - \theta_L/\theta_b)}{\sinh mL}$ (3.83)
D	Infinite fin ($L \rightarrow \infty$): $\theta(L) = 0$	e^{-mx} (3.84)	M (3.85)

$\theta \equiv T - T_\infty$ $\qquad m^2 \equiv hP/kA_c$

$\theta_b = \theta(0) = T_b - T_\infty$ $\qquad M \equiv \sqrt{hPkA_c}\,\theta_b$

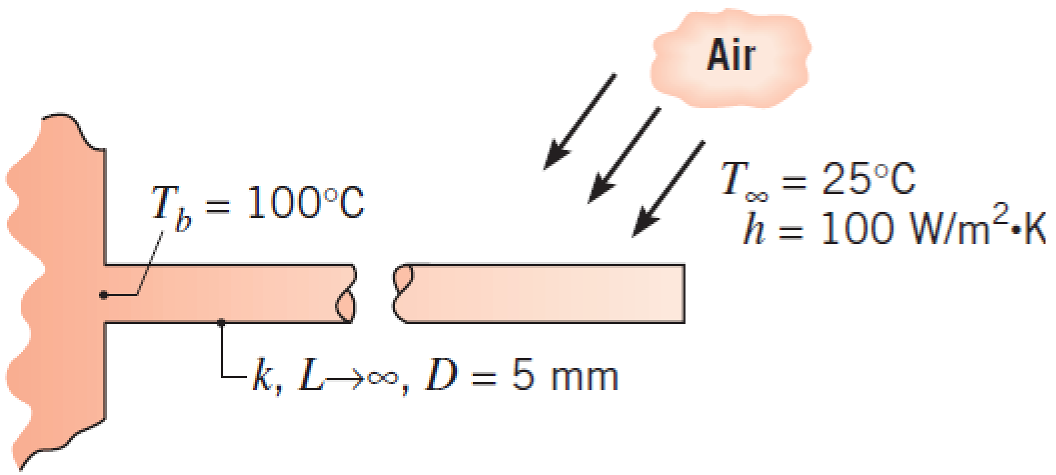
Heat Transfer from Extended Surfaces

2. 1. Fins of Uniform Cross-Sectional Area

Example-1

A very long rod 5 mm in diameter has one end maintained at 100 °C. The surface of the rod is exposed to ambient air at 25 °C with a convection heat transfer coefficient of 100 W/m^2 . K.

- *Determine the temperature distributions along rods constructed from pure copper, 2024 aluminum alloy, and type AISI 316 stainless steel.*
- *What are the corresponding heat losses from the rods?*



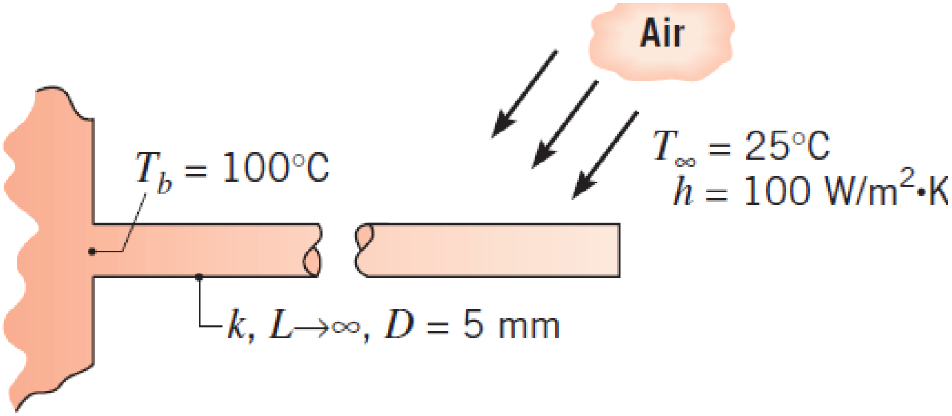
Heat Transfer from Extended Surfaces

2. 1. Fins of Uniform Cross-Sectional Area

Example-1

1. Steady-state conditions.
2. One-dimensional conduction along the rod.
3. Constant properties.
4. Negligible radiation exchange with surroundings.
5. Uniform heat transfer coefficient.
6. Infinitely long rod.

Properties: Table A.1, copper [$T = (T_b + T_\infty)/2 = 62.5°C \approx 335$ K]: $k = 398$ W/m·K. Table A.1, 2024 aluminum (335 K): $k = 180$ W/m·K. Table A.1, stainless steel, AISI 316 (335 K): $k = 14$ W/m·K.



Heat Transfer from Extended Surfaces

2. 1. Fins of Uniform Cross-Sectional Area

Example-1

TABLE 3.4 Temperature distribution and heat loss for fins of uniform cross section

Case	Tip Condition ($x = L$)	Temperature Distribution θ/θ_b		Fin Heat Transfer Rate q_f	
D	Infinite fin ($L \rightarrow \infty$): $\theta(L) = 0$	$\boxed{e^{-mx}}$	(3.84)	M	(3.85)

$\theta \equiv T - T_\infty$ $\qquad m^2 \equiv hP/kA_c$

$\theta_b = \theta(0) = T_b - T_\infty$ $\qquad M \equiv \sqrt{hPkA_c}\,\theta_b$

$$T = T_\infty + (T_b - T_\infty)e^{-mx}$$

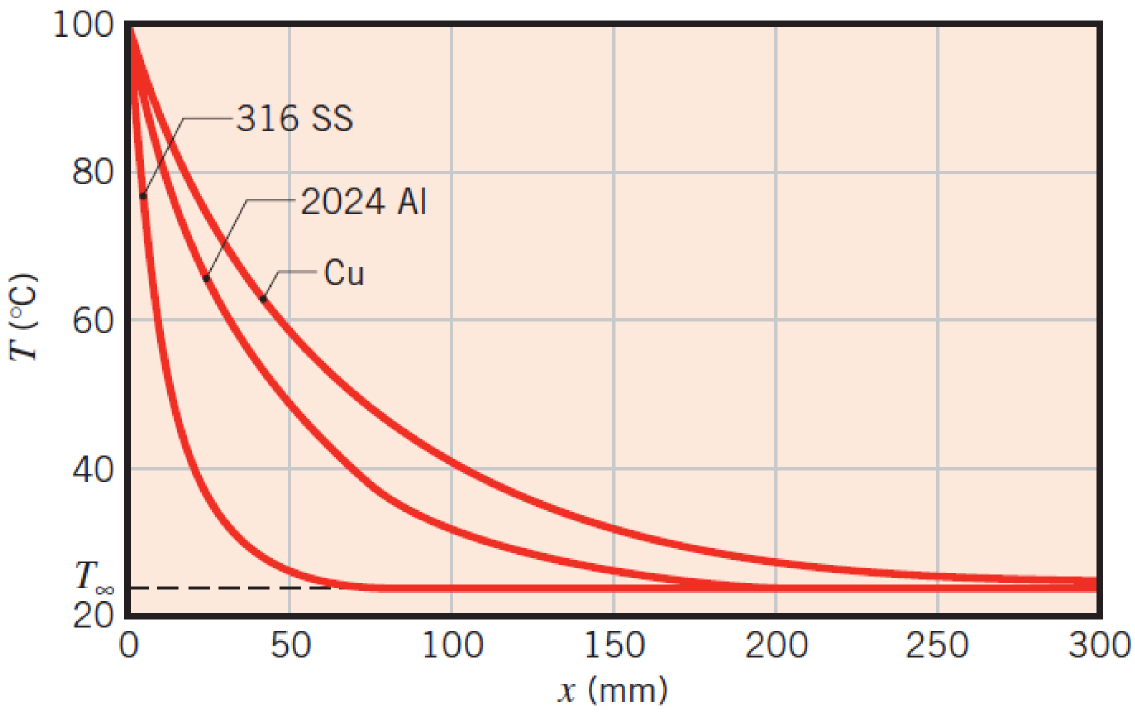
$$m = (hP/kA_c)^{1/2} = (4h/kD)^{1/2}$$

Heat Transfer from Extended Surfaces

2. 1. Fins of Uniform Cross-Sectional Area

Example-1

$$T = T_\infty + (T_b - T_\infty)e^{-mx} \qquad m = (hP/kA_c)^{1/2} = (4h/kD)^{1/2}$$



k_{Cu} = 398 W/m.K : m=14 m^{-1}
k_{Al} = 180 W/m.K : m=21 m^{-1}
k_{Fe} = 14 W/m.K : m=75 m^{-1}

Heat Transfer from Extended Surfaces

2. 1. Fins of Uniform Cross-Sectional Area

Example-1

$$q_f = \sqrt{hPkA_c}\,\theta_b$$

Hence for copper,

$$q_f = \left[100\ \mathrm{W/m^2 \cdot K} \times \pi \times 0.005\ \mathrm{m} \right.$$

$$\left. \times\ 398\ \mathrm{W/m \cdot K} \times \frac{\pi}{4}(0.005\ \mathrm{m})^2\right]^{1/2} (100 - 25)^\circ\mathrm{C}$$

$$=\ 8.3\ \mathrm{W}$$

$q_{Al} = 5.6$ W
$q_{Fe} = 1.6$ W

Heat Transfer from Extended Surfaces

2. 1. Fins of Uniform Cross-Sectional Area

Example-1

- Estimate how long the rods must be for the assumption of *infinite length* to yield an accurate estimate of the heat loss.

In this case the B boundary condition can be used to calculate the length at which we have the infinite condition (no heat transfer at the tip of the very long fin).

Case	Tip Condition ($x = L$)	Temperature Distribution θ/θ_b		Fin Heat Transfer Rate q_f	
B	Adiabatic: $d\theta/dx\vert_{x=L} = 0$	$\dfrac{\cosh m(L-x)}{\cosh mL}$	(3.80)	$M \tanh mL$	(3.81)
D	Infinite fin ($L \rightarrow \infty$): $\theta(L) = 0$	e^{-mx}	(3.84)	M	(3.85)

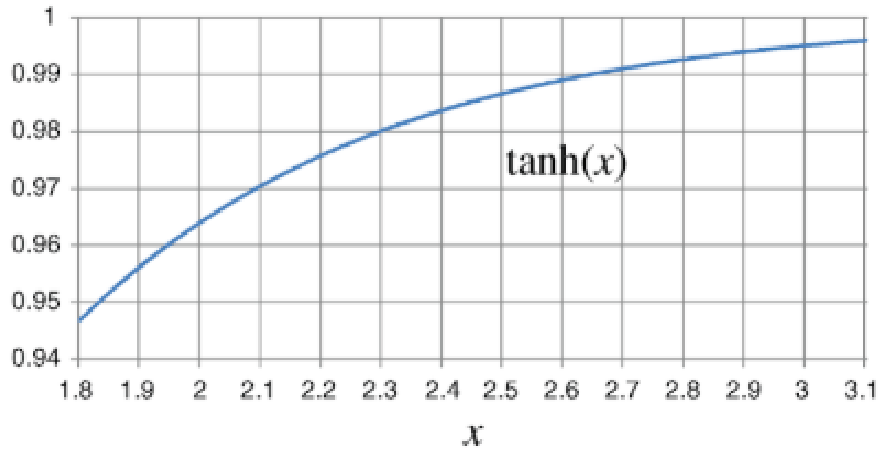
$\theta \equiv T - T_\infty$ $\quad m^2 \equiv hP/kA_c$

$\theta_b = \theta(0) = T_b - T_\infty$ $\quad M \equiv \sqrt{hPkA_c}\,\theta_b$

Heat Transfer from Extended Surfaces

2. 1. Fins of Uniform Cross-Sectional Area

Example-1



$$L_\infty \equiv \frac{2.65}{m} = 2.65\left(\frac{kA_c}{hP}\right)^{1/2}$$

For copper,

$$L_\infty = 2.65\left[\frac{398 \text{ W/m}\cdot\text{K} \times (\pi/4)(0.005 \text{ m})^2}{100 \text{ W/m}^2\cdot\text{K} \times \pi(0.005 \text{ m})}\right]^{1/2} = 0.19 \text{ m}$$