Introduction to Robotics

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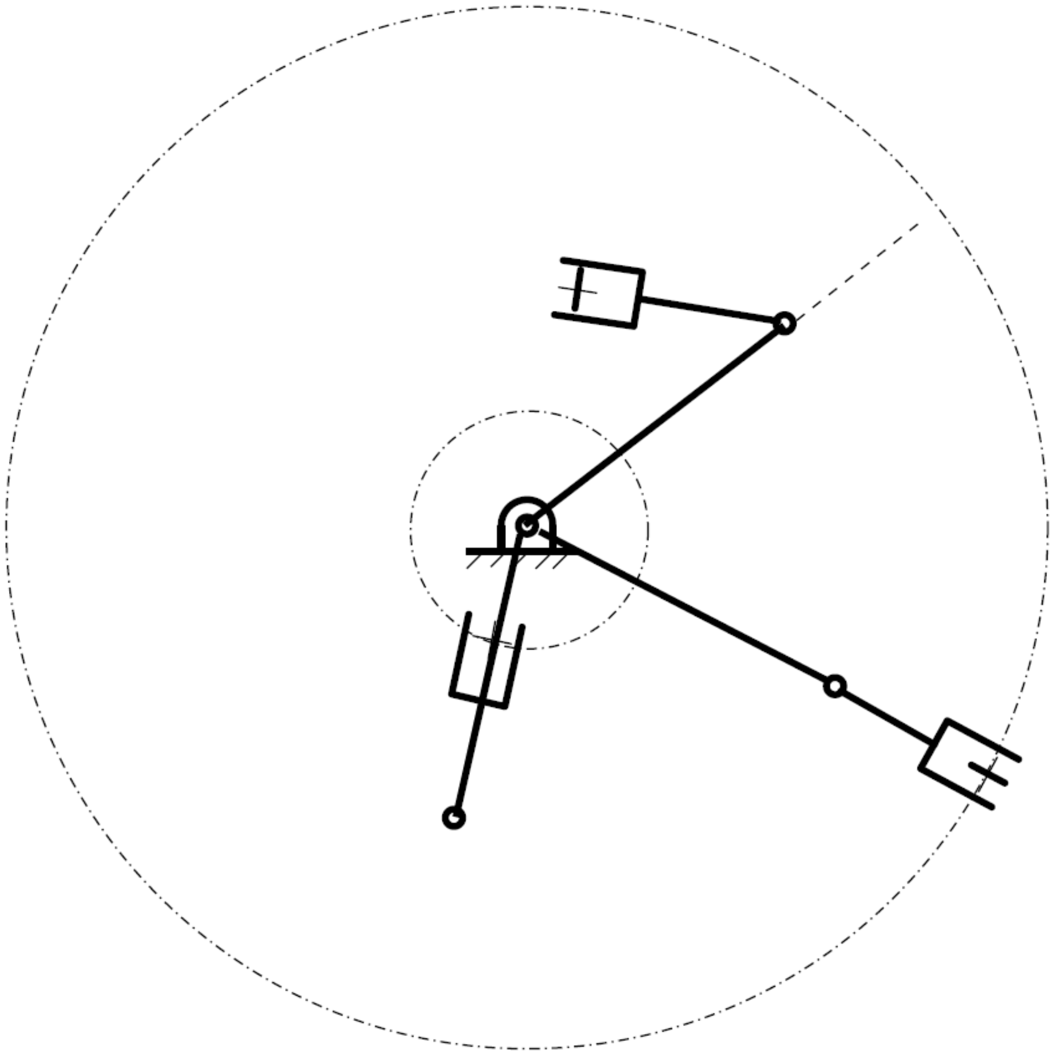
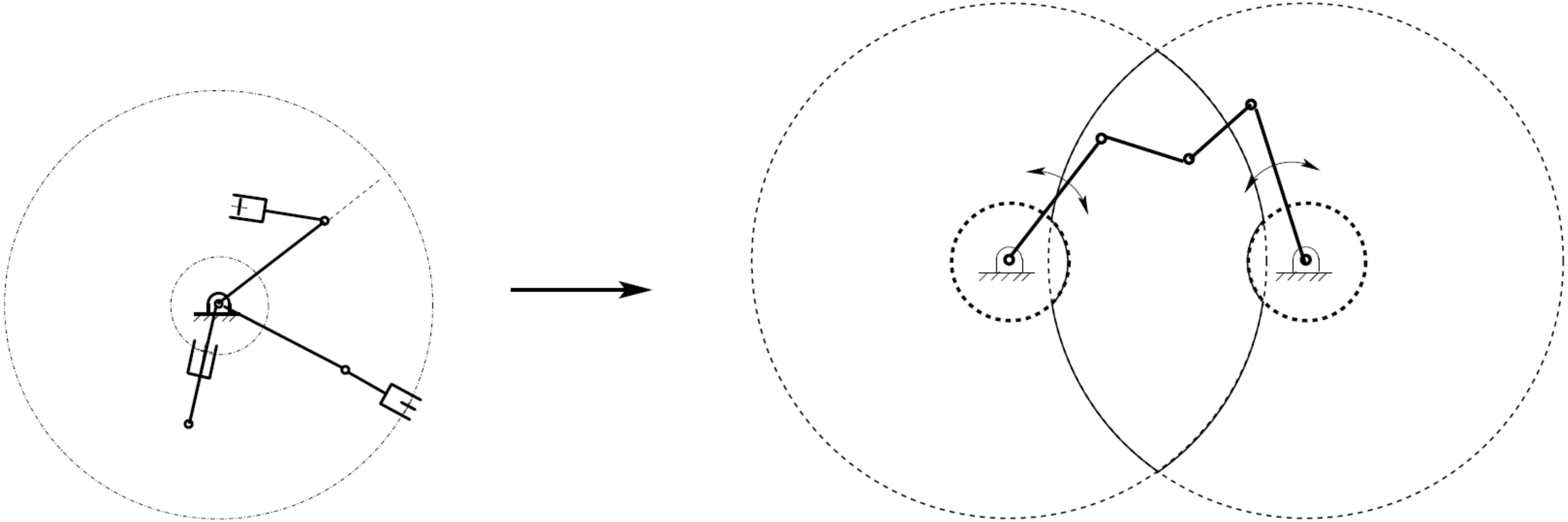
A simple serial manipulator: planar 2-R



A comparison with serial manipulators




Another example of a serial singularity
(one or both arms are fully extended)
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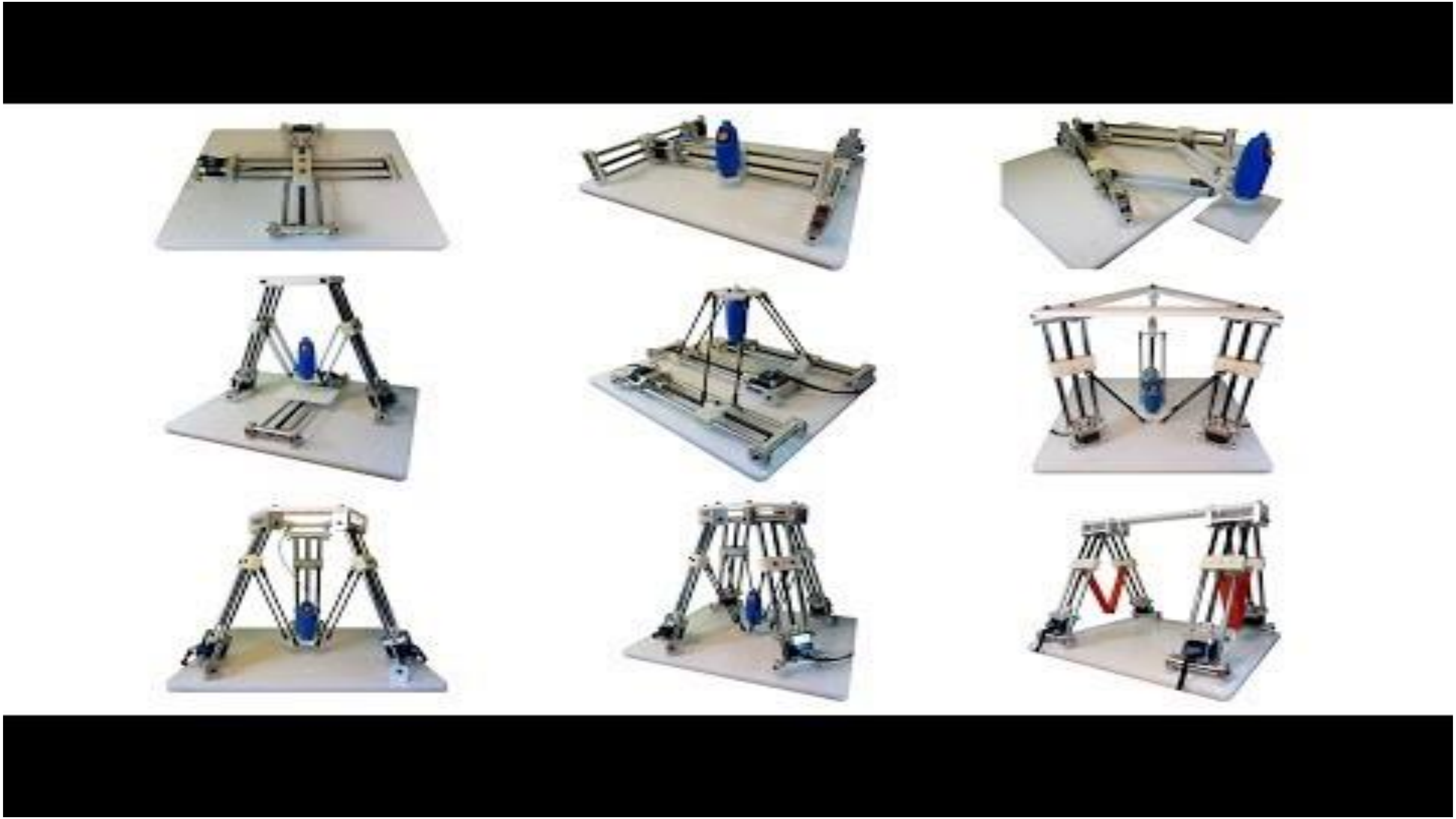
High-Speed Mode

Basic features of a parallel manipulator

- “Parallel manipulator” is a short form of “in-parallel actuated manipulator”
- Defining features:
  - More than one point on the manipulator is fixed, i.e., the links form one or more loops between the fixed points on the fixed base platform.
  - Typically, more than one limb meet at an end-effector, which is generally termed as the moving platform.
  - The manipulator has more links than actuators; these additional links are called “passive”, and their positions are determined by solving the loop-closure equations.



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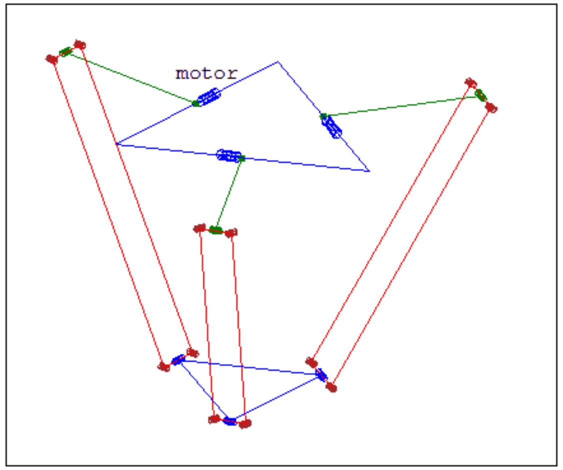
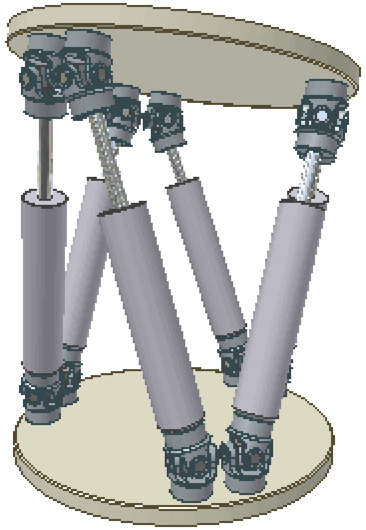
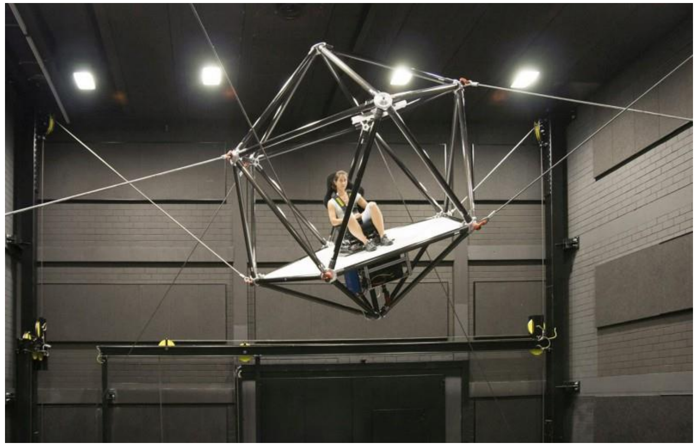
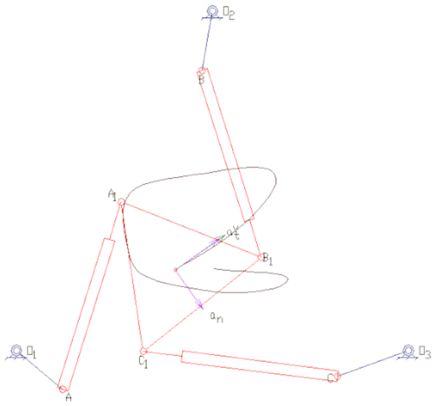
Basic Types of Parallel Manipulators

- Delta Robot
- Gough-Stewart Robot
- Planner Parallel Robots
- Wire Driven Robots

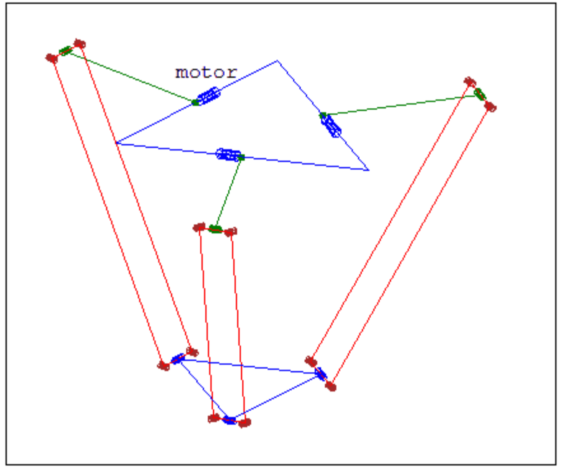




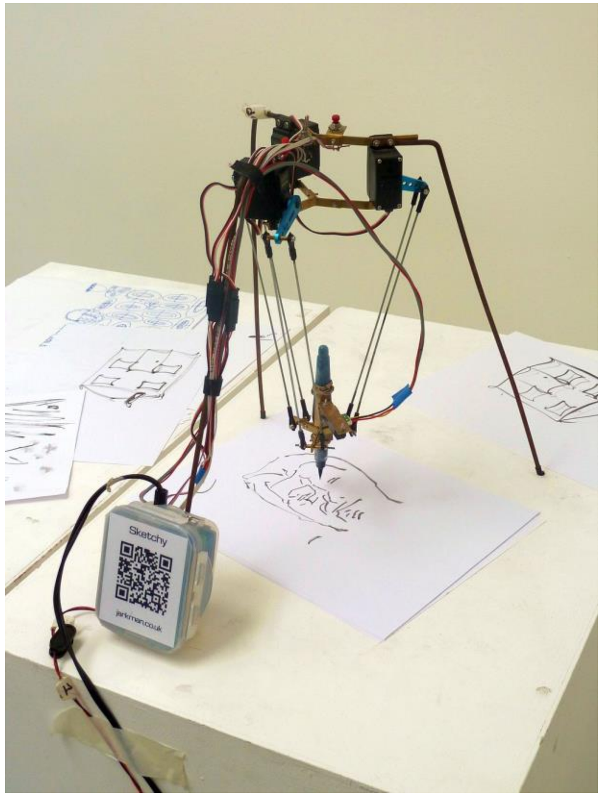




Delta Robot

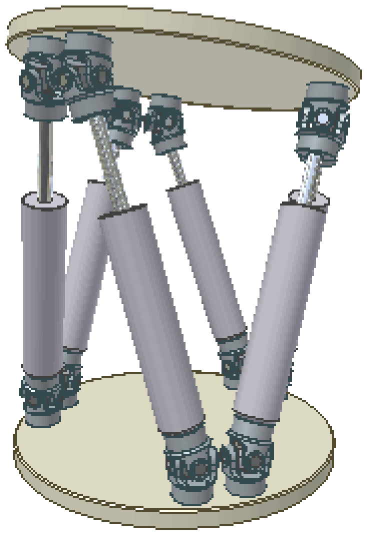


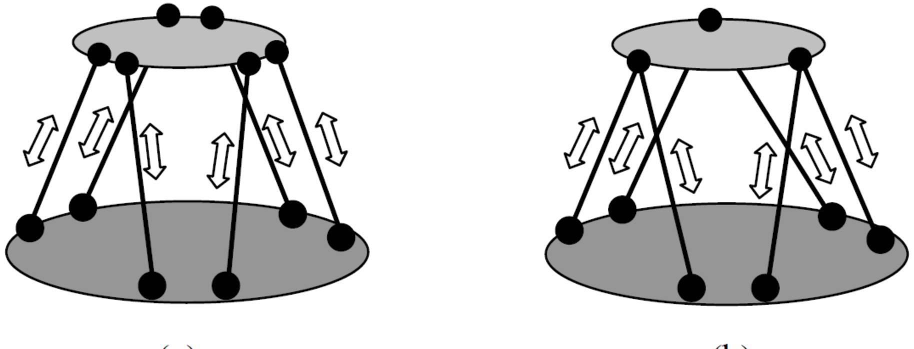
- The delta robot (a parallel arm robot) was invented in the early 1980s by a research team led by professor Reymond Clavel at the École Polytechnique Fédérale de Lausanne (EPFL, Switzerland
- In 1987, the Swiss company Demaurex purchased a license for the delta robot and started the production of delta robots for the packaging industry.
- In 1991 Reymond Clavel presented his doctoral thesis 'Conception d'un robot parallèle rapide à 4 degrés de liberté', and received the golden robot award in 1999 for his work and development of the delta robot.
- Also in 1999, ABB Flexible Automation started selling its delta robot, the FlexPicker. By the end of 1999 delta robots were also sold by Sigpack Systems.
- In 2017 Harvard's Microrobotics Lab researcher Hayley McClintock miniaturized it with piezoelectric actuators to 0.43 grams for 15 mm x 15 mm x 20 mm, capable of moving a 1.3g payload around a 7 cubic millimeter workspace with a 5 micrometers precision, reaching 0.45 m/s speeds with 215 m/s² accelerations and repeating patterns at 75 Hz.



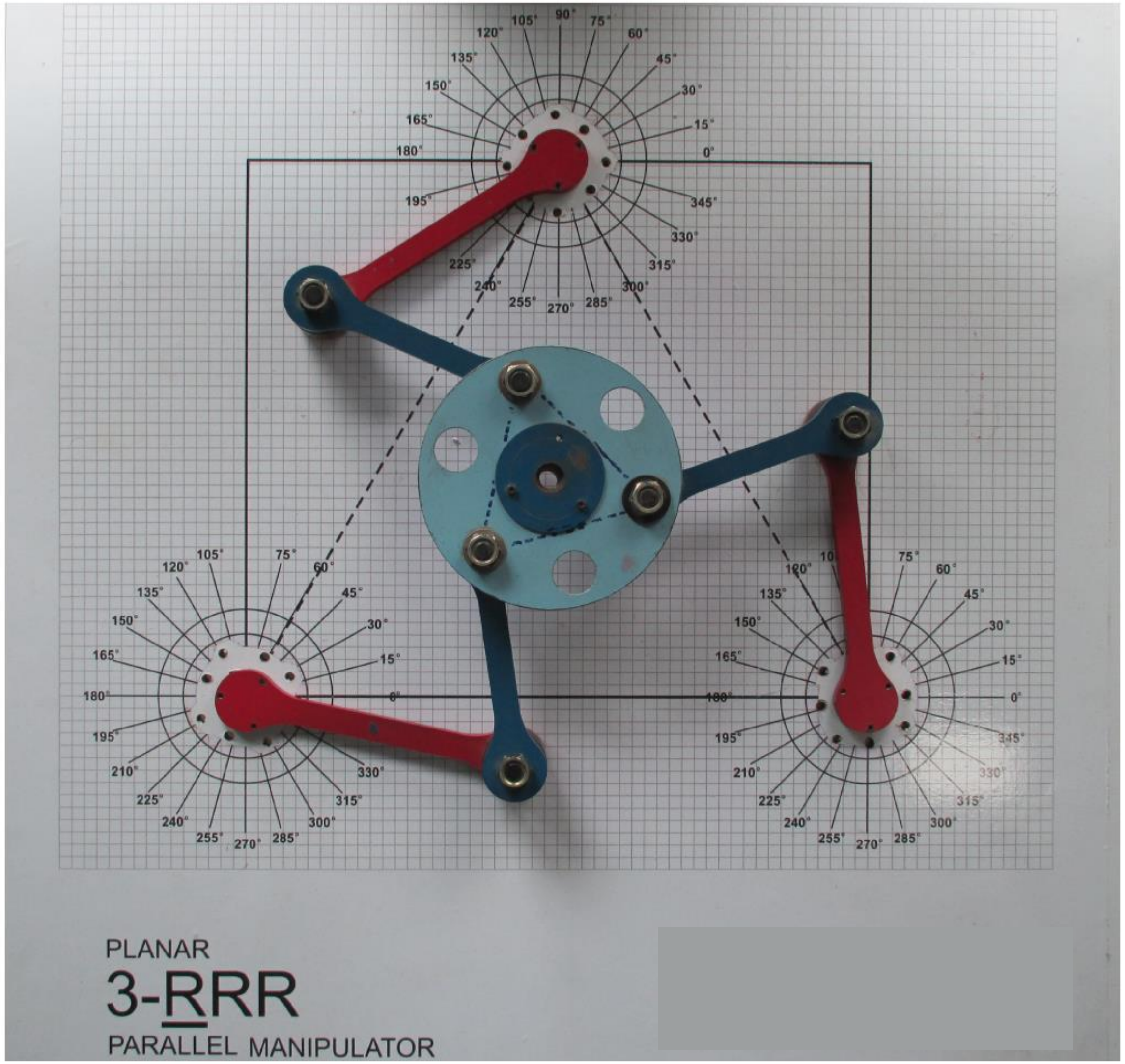
Gough-Stewart Robot

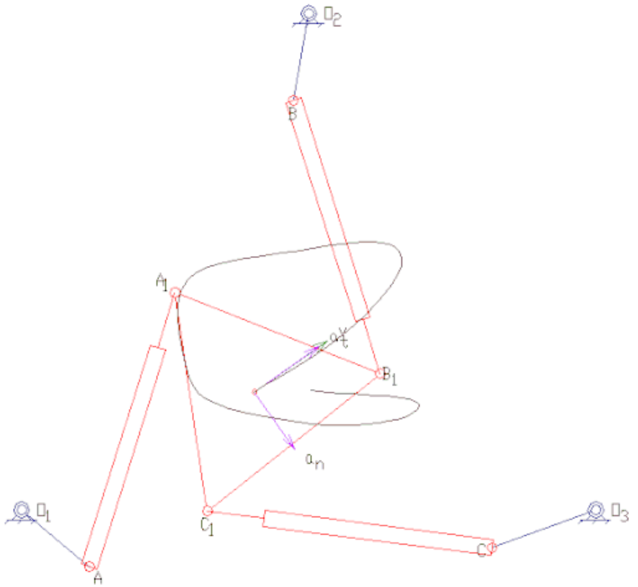
- A **Gough-Stewart platform** is a type of parallel robot that has six prismatic actuators, commonly hydraulic jacks or electric actuators, attached in pairs to three positions on the platform's baseplate, crossing over to three mounting points on a top plate.
- Devices placed on the top plate can be moved in the six degrees of freedom in which it is possible for a freely-suspended body to move.
- These are the three linear movements x, y, z and three rotational
    - lateral, longitudinal and vertical
    - pitch, roll, & yaw.



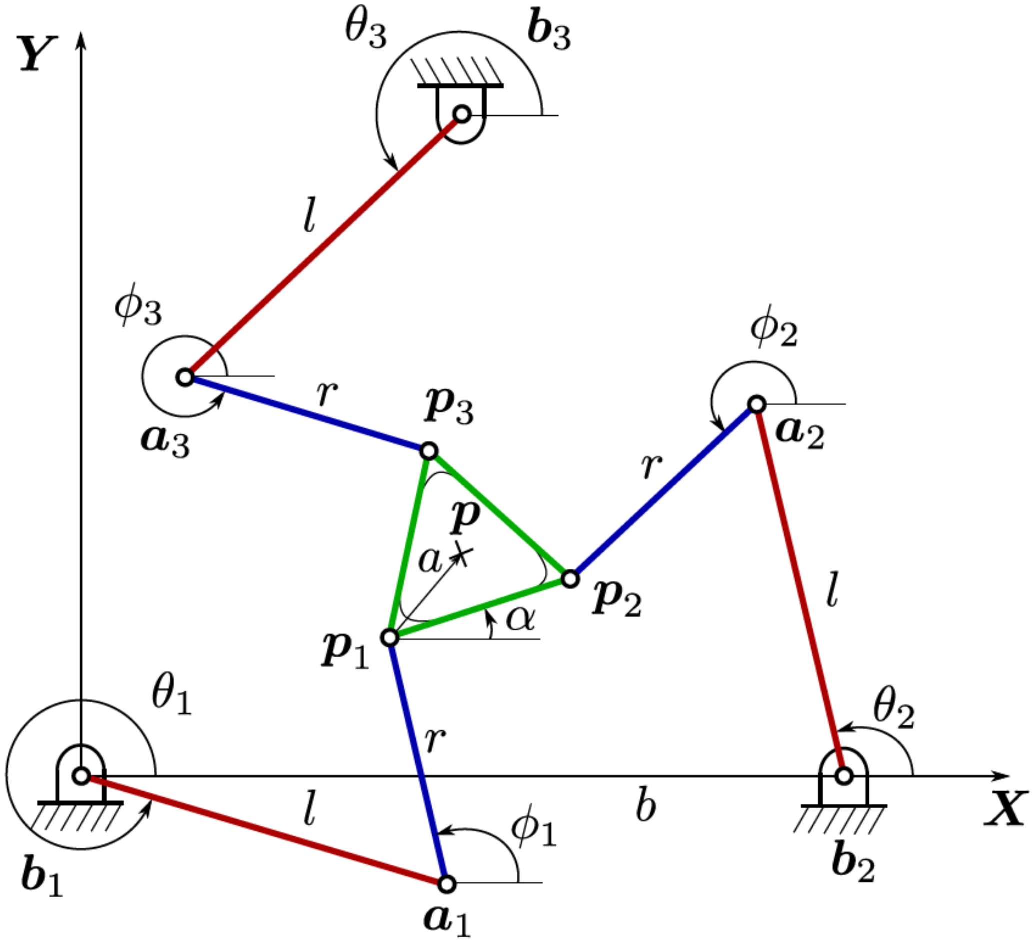


Example: The 3-$\underline{R}$RR planar parallel manipulator





Kinematic model



Fixed base: $\Delta \boldsymbol{b}_1 \boldsymbol{b}_2 \boldsymbol{b}_3$

Moving platform: $\Delta \boldsymbol{p}_1 \boldsymbol{p}_2 \boldsymbol{p}_3$

Kinematic loops:

$\boldsymbol{b}_1 - \boldsymbol{p}_1 - \boldsymbol{p}_2 - \boldsymbol{b}_2 - \boldsymbol{b}_1,$

$\boldsymbol{b}_2 - \boldsymbol{p}_2 - \boldsymbol{p}_3 - \boldsymbol{b}_3 - \boldsymbol{b}_2,$

$\boldsymbol{b}_3 - \boldsymbol{p}_3 - \boldsymbol{p}_1 - \boldsymbol{b}_1 - \boldsymbol{b}_3$

Active variables:

$\boldsymbol{\theta} = (\theta_1, \theta_2, \theta_3)^T$

Passive variables:

$\boldsymbol{\phi} = (\phi_1, \phi_2, \phi_3)^T$

Advantages of parallel manipulators

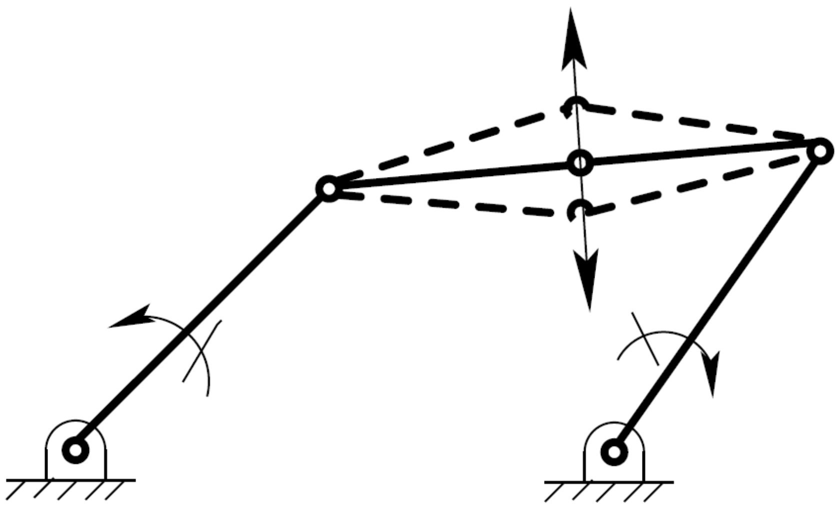
- High load capacity
- High stiffness
- High accuracy
- High speed

Disadvantages of parallel manipulators

- Small workspaces, with complicated geometry
- Singularities inside the workspace
- Complicated kinematics
- Frequent occurrences of link interference

Locking/gain-type singularities in parallel manipulators

Singularities (gain-type) occur *inside* the workspace, *locking* the mechanism, and leading to a *gain of uncontrollable DoF* at the same time!



Forward kinematics of parallel manipulators

- Usually requires the solution of non-linear
  - i.e., trigonometric/algebraic equations
- Generally takes a significant amount of effort to reach at the final result
  - a single univariate polynomial equation
- The polynomial can be of moderate to high degree:
  - 5-bar planar manipulator: 2 degrees
  - 3-RRR planar manipulator: 6 degrees
  - 3-RPS spatial manipulator: 2~8 degrees
  - MaPaMan spatial manipulator: 16 degrees
  - Stewart platform manipulator: 40 degrees

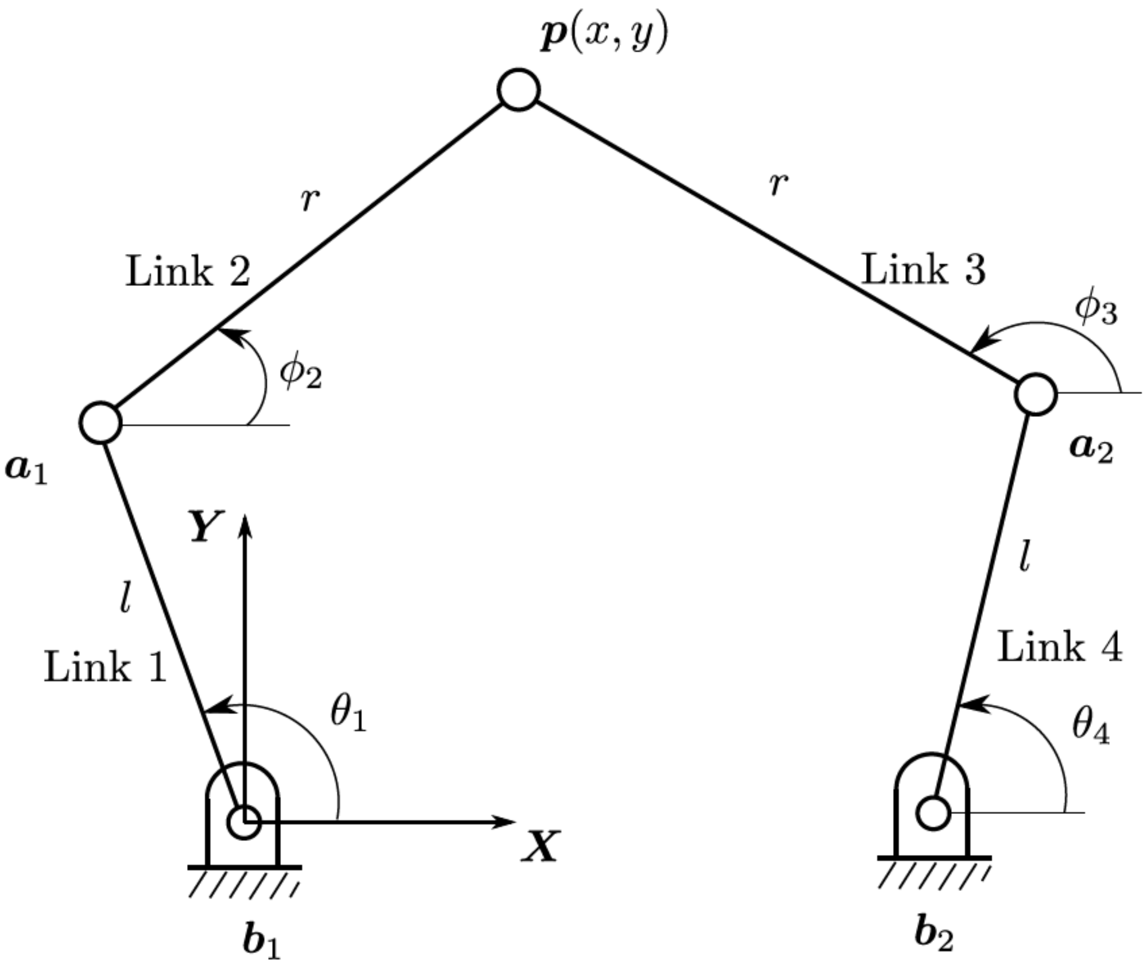
Additional issues: singularities

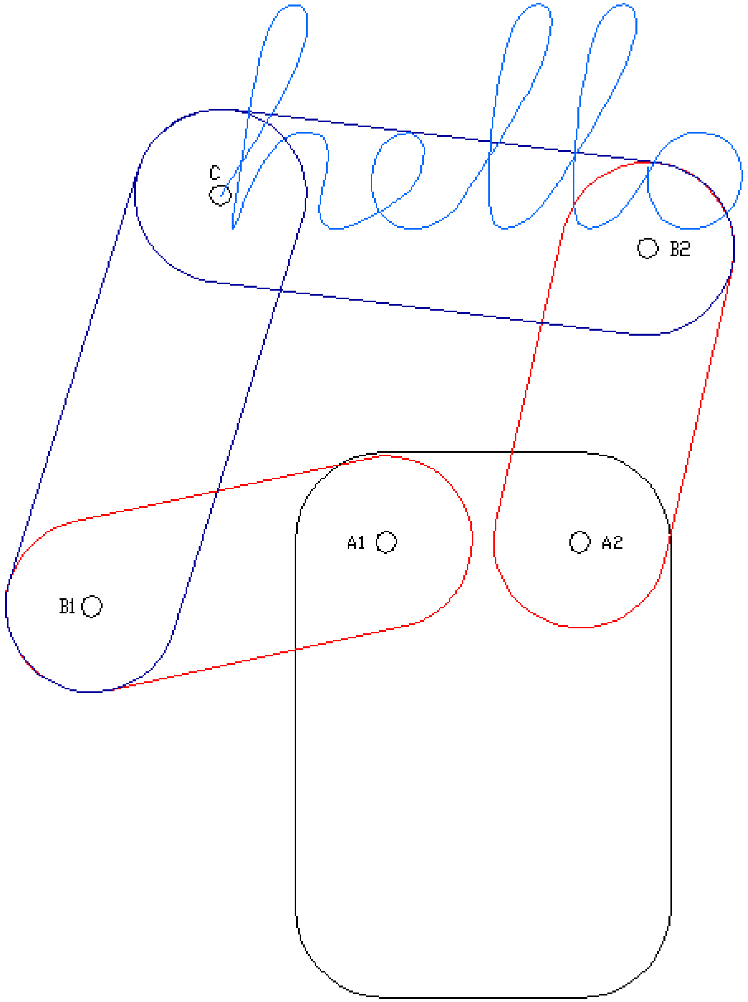
- Singularities occur whenever one or more pair(s) of the solutions merge.
- Analysis of singularities is much harder
  - leads to polynomials of much higher degrees!
  - E.g., 3-RRR, singularities occur on a degree 42 curve in the plane of the manipulator!
- Even if the results are obtained, they cannot always be interpreted geometrically.

Geometric approach to position analysis

- Offers a visual interpretation of the solutions
- Often leads to an intuitive understanding of the singularities
- Leads to a mathematical formulation in terms of algebraic/trigonometric equations
- Needs a bit of geometric imagination

Position analysis of the planar 5-bar manipulator





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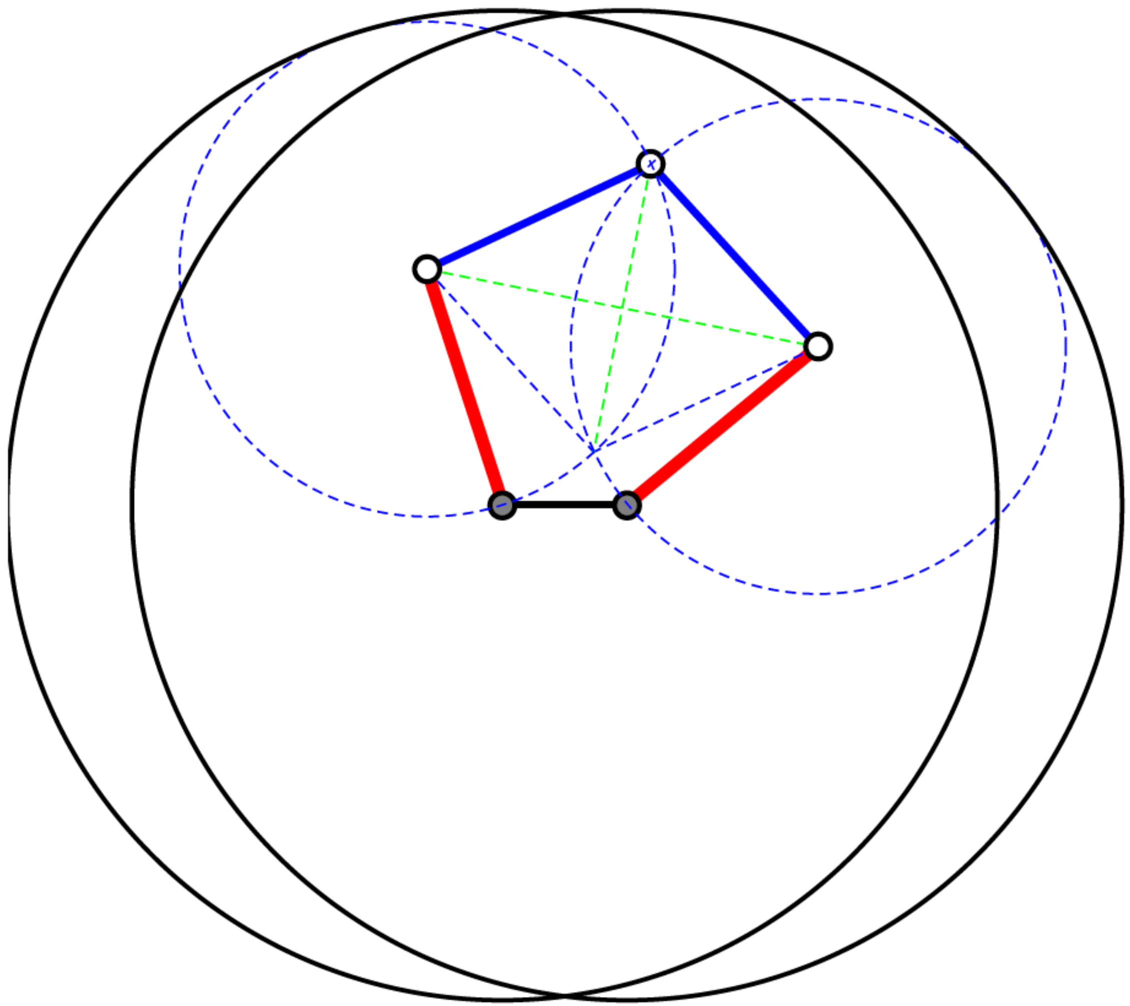
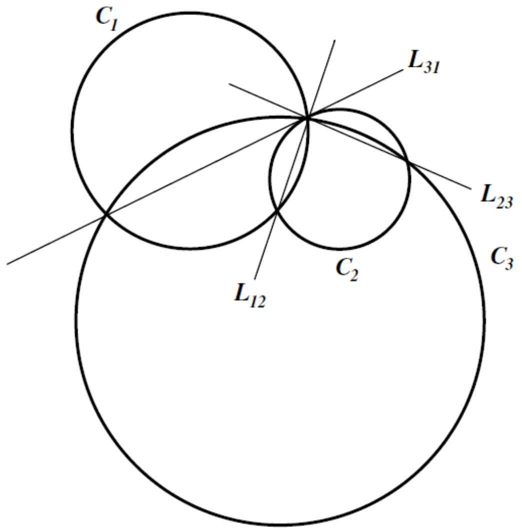
Two circles of the planar 5-bar manipulator



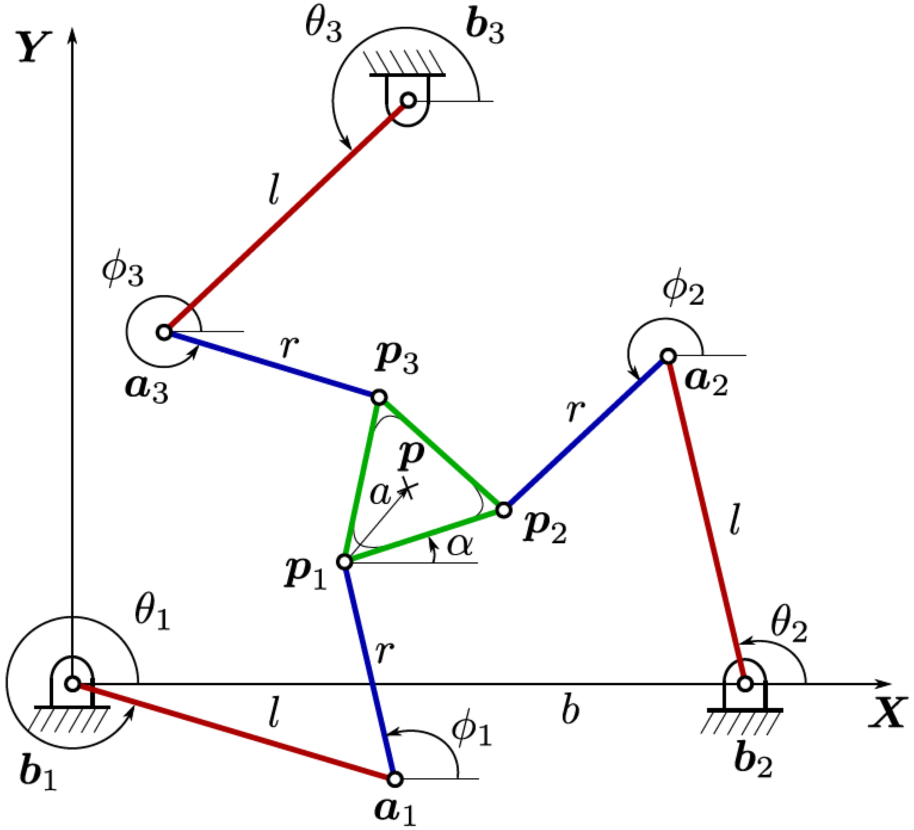
Intersection of three circles in the plane



Three circles in a plane meet at a point if their pair-wise common chords meet at a (i.e., at the same) point:

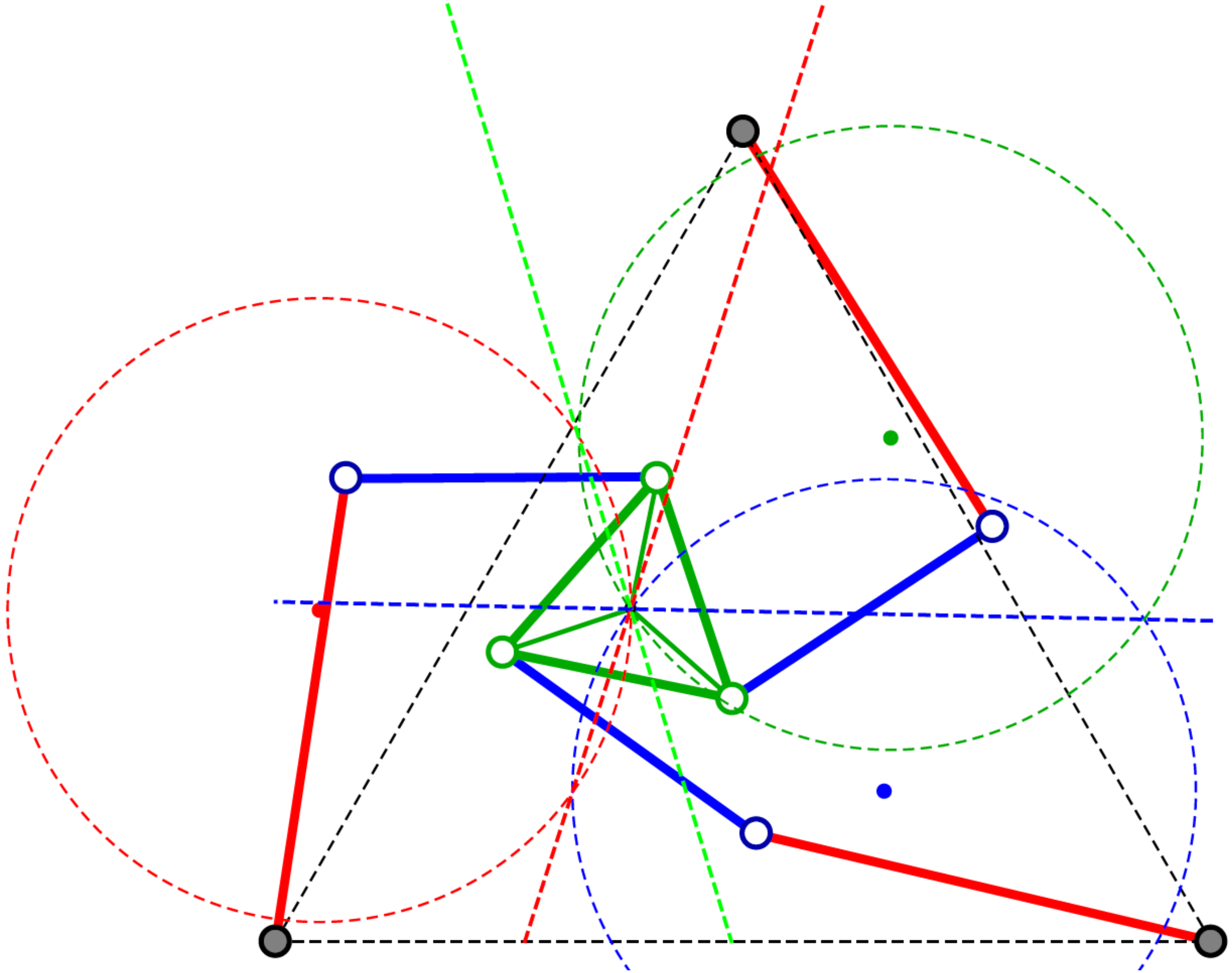
$$\Rightarrow \langle C_1, C_2, C_3 \rangle = \langle L_{12}, L_{23}, L_{31} \rangle \,.$$

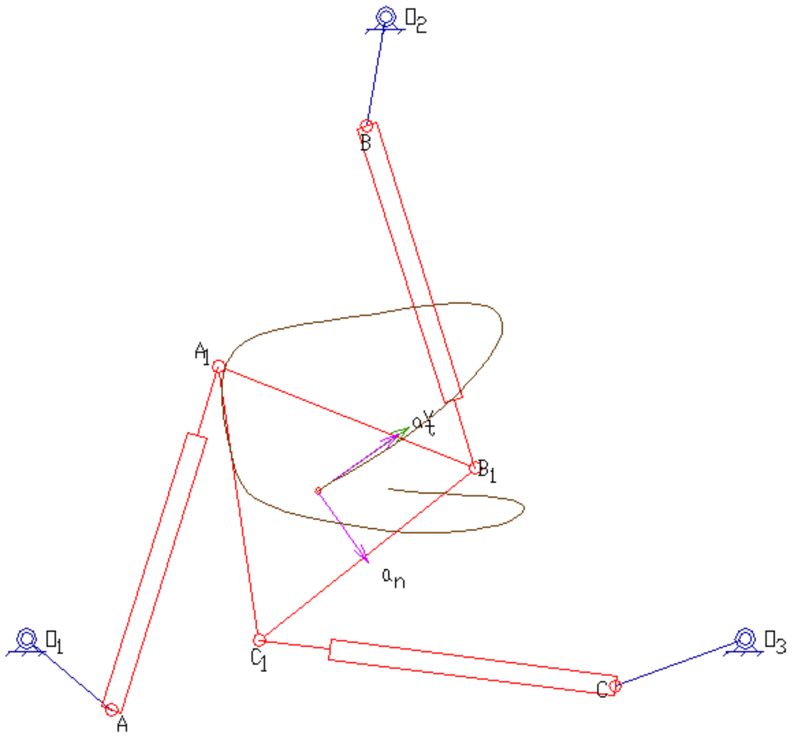
Position analysis of the 3-RRR manipulator



Three circles: $(\boldsymbol{p}_i - \boldsymbol{a}_i)^\top (\boldsymbol{p}_i - \boldsymbol{a}_i) - r^2 = 0,\ i = 1, 2, 3.$

Three circles of the 3-RRR manipulator





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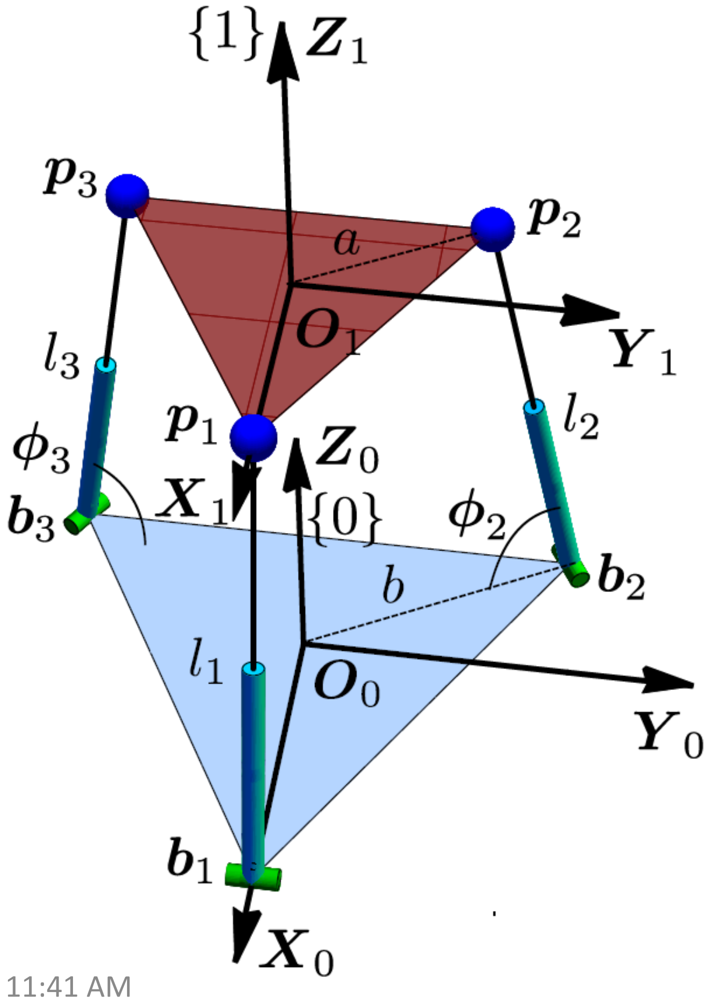
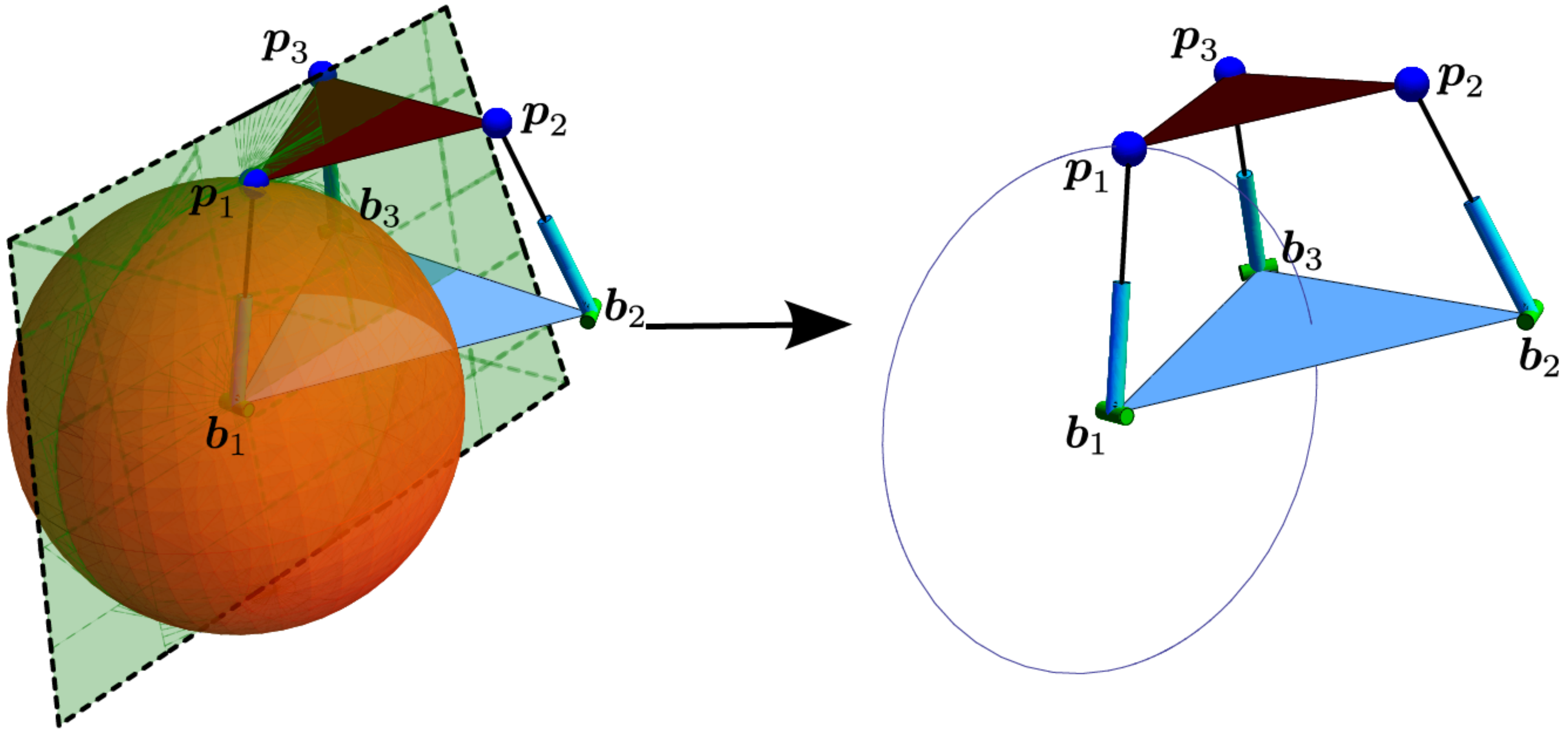
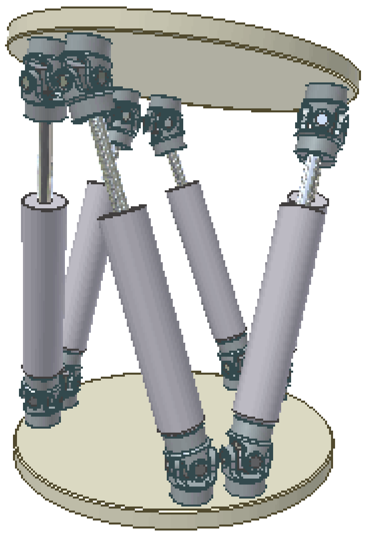
Geometry of the 3-RPS manipulator



- 3-DoF: roll, pitch and heave
- Moving platform: $\Delta p_1 p_2 p_3$
- Base platform: $\Delta b_1 b_2 b_3$
- Active variables: $\theta = (l_1, l_2, l_3)^\top$
- Passive variables: $\phi = (\phi_1, \phi_2, \phi_3)^\top$

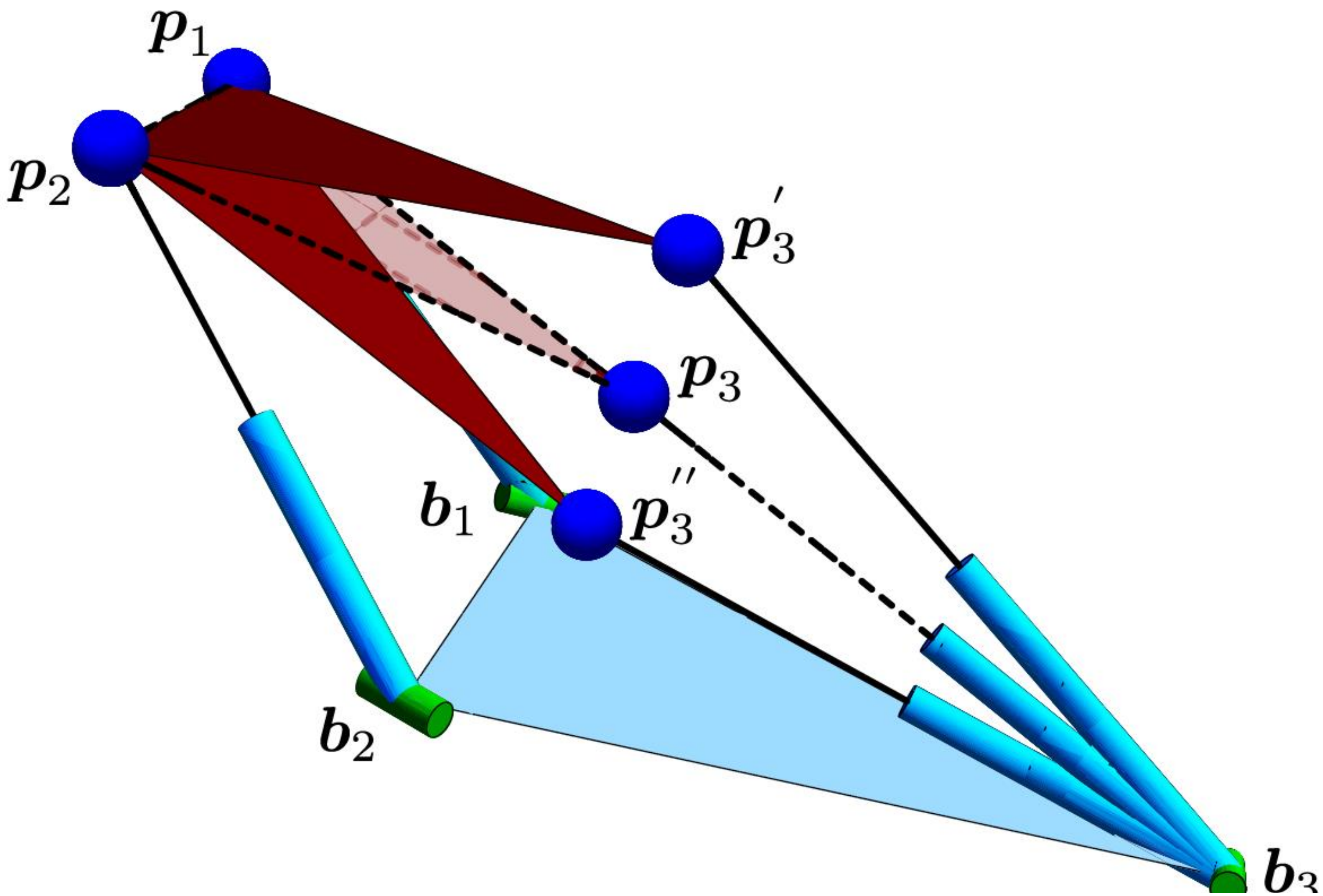
Circles of constraint on each leg



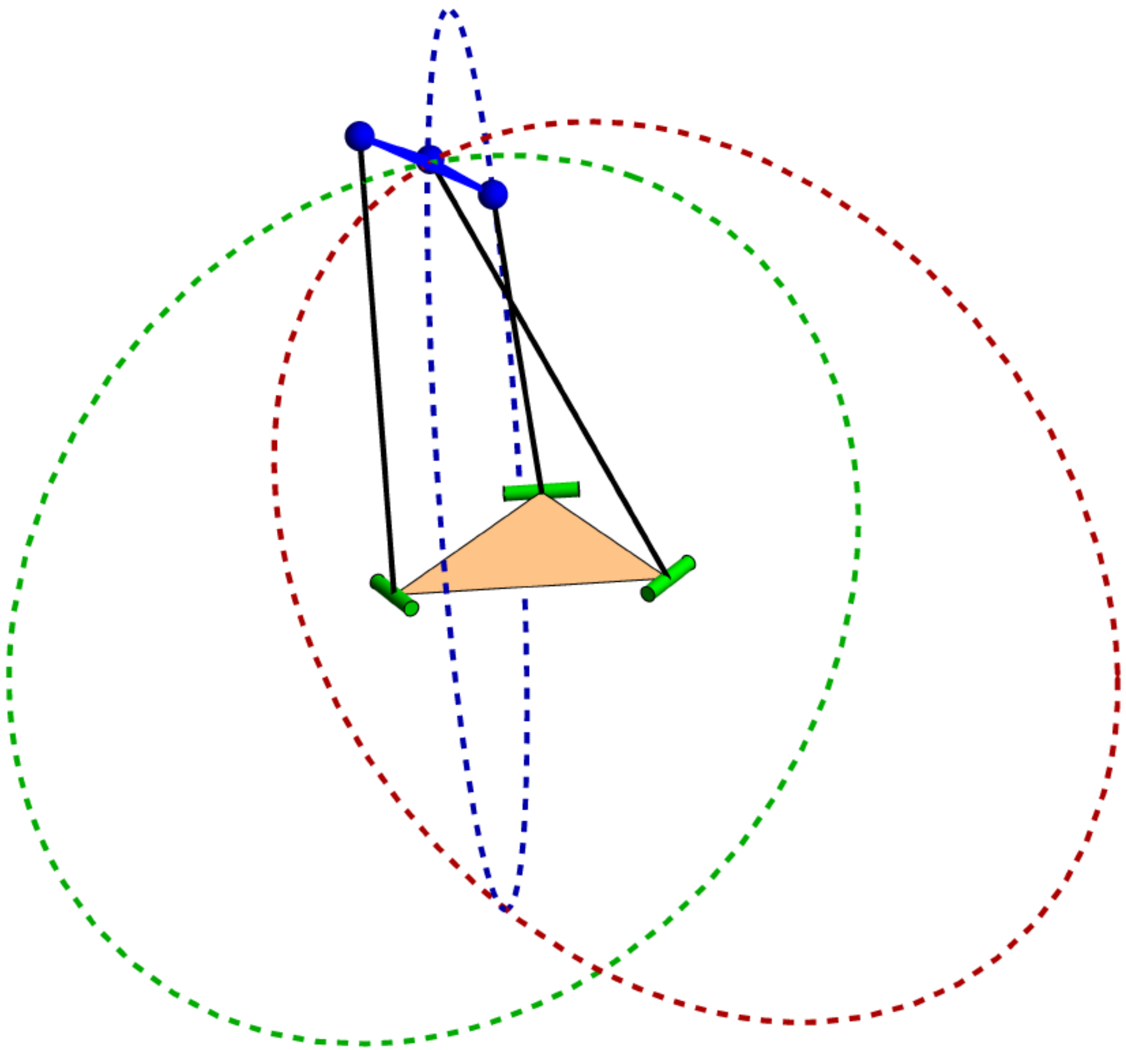


Spherical constraint: $(\boldsymbol{p}_i - \boldsymbol{b}_i)^\top(\boldsymbol{p}_i - \boldsymbol{b}_i) = l_i^2,\ i = 1, 2, 3.$

Operation modes and singularities



Position analysis of the 3-R<u>P</u>S manipulator: 3 circles again!



Position analysis of parallel manipulators can be reduced to simple geometric problems.

In many cases, it demands the knowledge of geometry not above the level of high-school!

It is not just about forming and solving equations – it is interesting, and beautiful!

Forward kinematics - parallel robots

A parallel mechanism is symmetrical if the

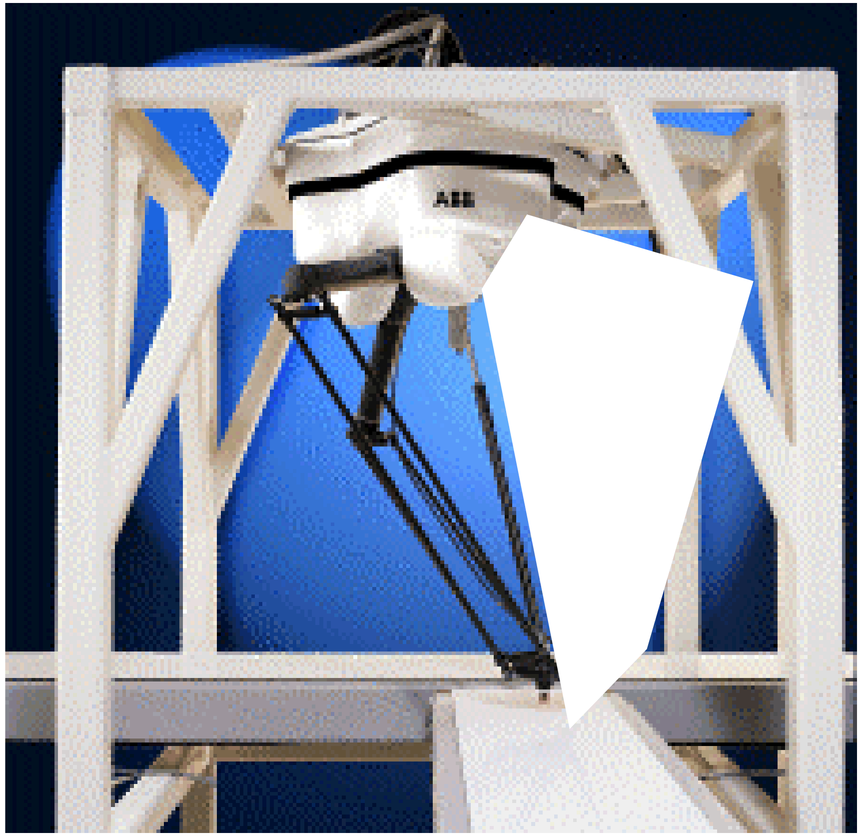
- number of limbs is equal to the number of degrees-of-freedom of the moving platform
- joint type and joint sequence in each limb are the same
- number and location of the actuated joints are the same.

Otherwise, the mechanism is asymmetrical. We will examine the kinematics for symmetrical mechanisms.

Parallel robot definitions

limb = a serial combination links and joints between ground and the moving rigid platform

connectivity of a limb (C_k) = degrees-of-freedom associated with all joints in a limb



Connectivity equation

Observation of symmetrical mechanisms will establish that

$$\sum_{k=1}^{m} C_k = \sum_{i=1}^{j} f_i$$

where j is the number of joints in the parallel mechanism and m is the number of limbs.

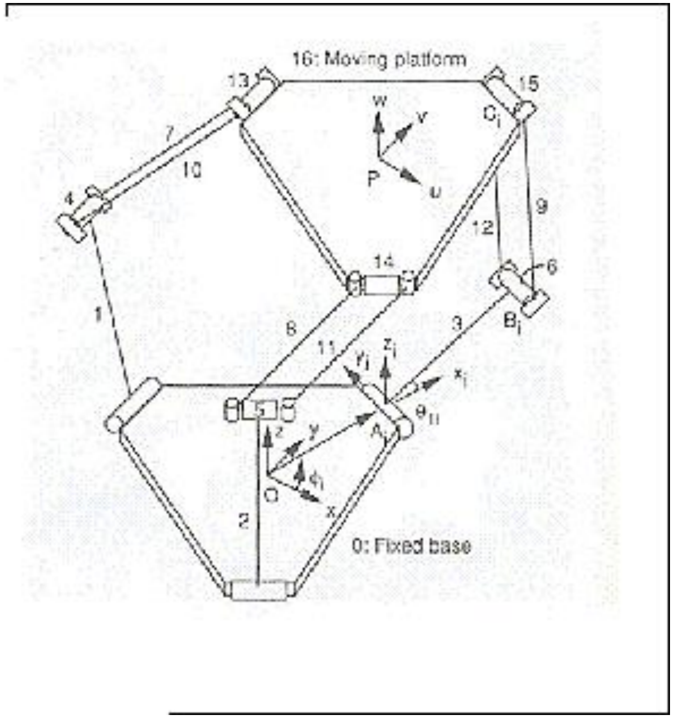
What is the connectivity of a limb of the picker robot?

Answer – 7!

Note that Grubler's Criterion does not readily apply to parallel robots because of joint redundancies.

Forward kinematics example

The course notes present the forward kinematics solution for the Maryland (or ABB picker) robot. This solution is also found in Tsai's text. It is complex and will not be discussed here, but you should review the solution to see how it is done.

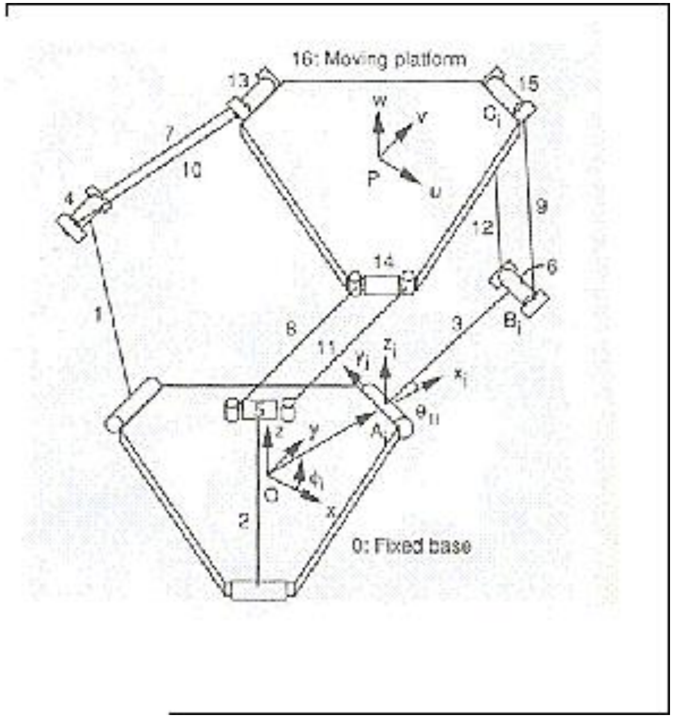


The reason for not examining this solution is found in the question:

How would you use a teach pendant to program this robot?

Program picker robot?

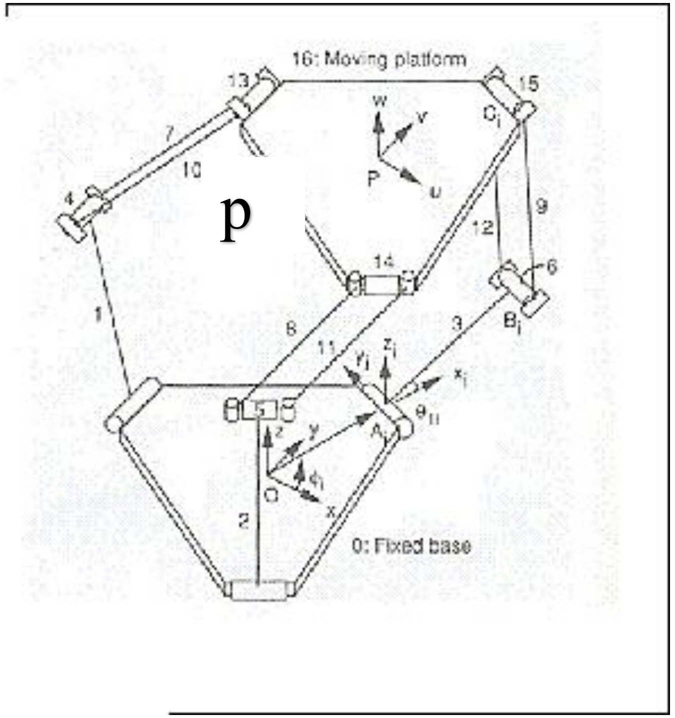
In reality you would probably not command the joints directly, but most likely command translations in the u, v, and w directions. Thus, you would not likely drive this robot using forward kinematics but only apply inverse kinematics.



Inverse kinematics for picker robot

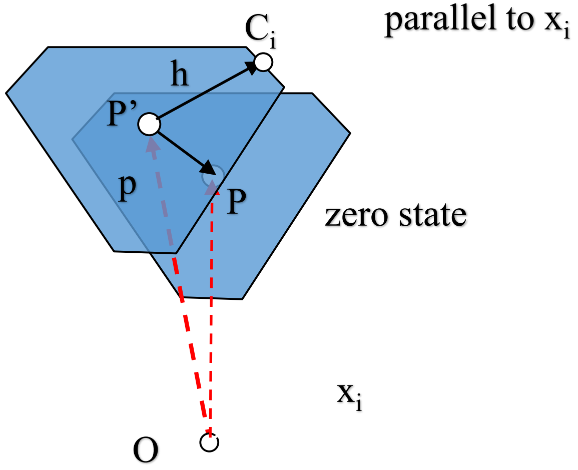
Assume that a desired position vector p given. Find the joint angles to place point P at p.

In reality, the gripper would not be located at P, but be attached to the moving platform. This is determined by gripper frame G relative to the platform coordinate axes. A target frame is specified as T. The frame for point P is determined from the fourth column of $T_p = TG^{-1}$. We designate this vector as p.



Inverse kinematics for picker robot

Given p we determine the location of point C_i. This is simple because the moving platform cannot rotate and thus the line between P and C_i translates only. Thus, given P (as determined by p) and the distance h, we can determine C_i as displaced from P by a vector of length h that is parallel to x_i.

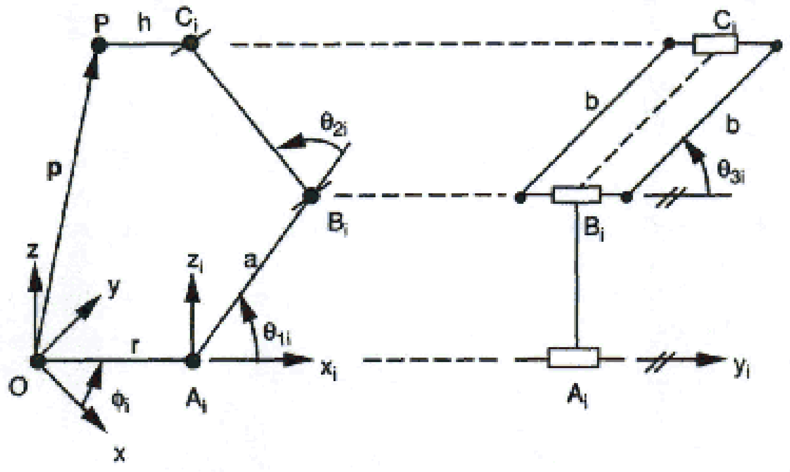


Inverse kinematics for picker robot

We can write a loop closure equation:

$$A_iB_i + B_iC_i = OP + PC_i - OA_i$$

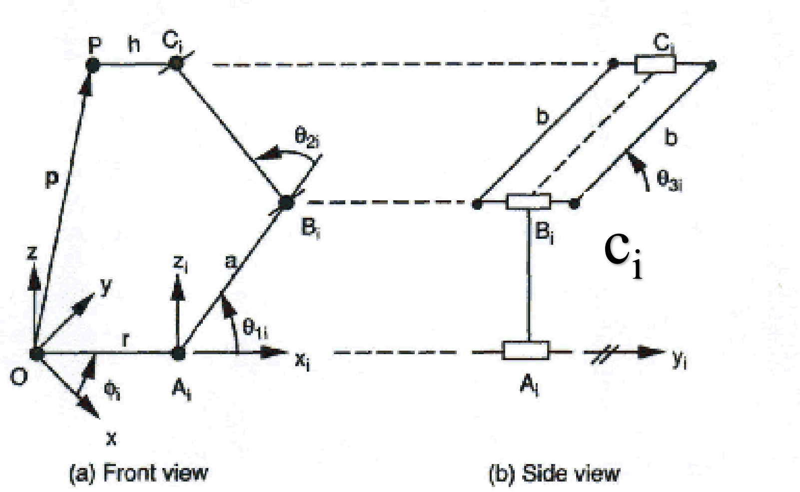
and express this equation in the limb i coordinate frame (x_i, y_i, z_i).



(a) Front view (b) Side view

Inverse kinematics for picker robot

Expressing the previous limb vector loop equation in the limb i coordinate frame, we get the matrix equations: in terms of the limb vector c shown by the green vector in the figure:



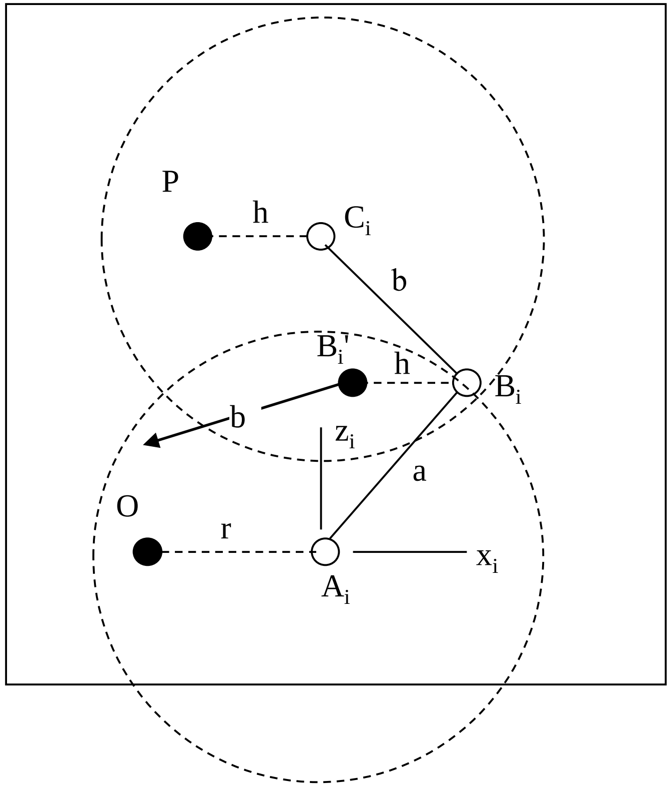
$$\begin{bmatrix} a\, c\theta_{1i} + b\, s\theta_{3i}\, c(\theta_{1i}+\theta_{2i}) \\ b\, c\theta_{3i} \\ a\, s\theta_{1i} + b\, s\theta_{3i}\, s(\theta_{1i}+\theta_{2i}) \end{bmatrix} = \begin{bmatrix} c_{xi} \\ c_{yi} \\ c_{zi} \end{bmatrix}$$

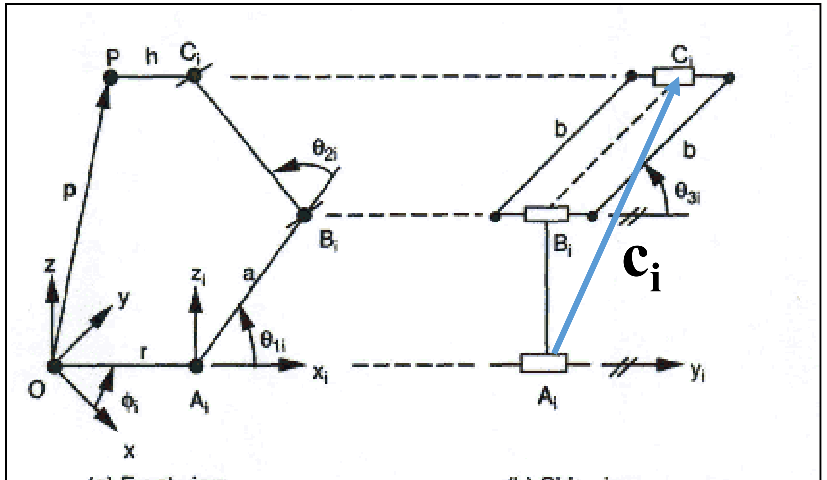
$$\begin{bmatrix} c_{xi} \\ c_{yi} \\ c_{zi} \end{bmatrix} = \begin{bmatrix} c\phi_i & s\phi_i & 0 \\ -s\phi_i & c\phi_i & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} + \begin{bmatrix} h-r \\ 0 \\ 0 \end{bmatrix}$$

Inverse kinematics for picker robot

The locus of motion of link B_iC_i is a sphere with center at C_i and radius b. From the previous matrix equation we can determine a solution for θ_{3i} as

$$\theta_{3i} = \cos^{-1}(c_{yi}/b)$$





(a) Front view
(b) Side view

Inverse kinematics for picker robot

Given θ_{3i} we can determine an equation for θ_{2i} by summing the squares of the matrix equation to get

$$2ab\, s\theta_{3i}\, c\theta_{2i} + a^2 + b^2 = c_{xi}^2 + c_{yi}^2 + c_{zi}^2$$

which leads to a solution for θ_{2i} as

$$\theta_{2i} = \cos^{-1}(\kappa)$$

where $\kappa = (c_{xi}^2 + c_{yi}^2 + c_{zi}^2 - a^2 - b^2)/(2ab\, s\theta_{3i})$. Physically, we can determine two solutions for θ_{2i} ("+" angle and "-" angle similar to elbow up/down case).

Inverse kinematics for picker robot

The two solutions for θ_{1i} can be determined from the matrix equation by expanding the double angle formulas, solving for the sine and cosine of θ_{1i} and then using the atan2 function to get θ_{1i}.

$$\begin{bmatrix} a\, c\theta_{1i} + b\, s\theta_{3i}\, c(\theta_{1i} + \theta_{2i}) \\ b\, c\theta_{3i} \\ a\, s\theta_{1i} + b\, s\theta_{3i}\, s(\theta_{1i} + \theta_{2i}) \end{bmatrix} = \begin{bmatrix} c_{xi} \\ c_{yi} \\ c_{zi} \end{bmatrix}$$

Inverse kinematics for picker robot

An alternative solution is to sum the squares of the 1st and 3rd equations after rearranging them to this form (θ_{3i} now known):

$$a\, c\theta_{1i} - c_{xi} = b\, s\theta_{3i}\, c(\theta_{1i} + \theta_{21})$$
$$a\, s\theta_{1i} - c_{zi} = b\, s\theta_{3i}\, s(\theta_{1i} + \theta_{21})$$

leading to:

$$c_{xi}\, c\theta_{1i} + c_{zi}\, s\theta_{1i} = (a^2 + c_{xi}{}^2 + c_{zi}{}^2 - b^2\, s^2\theta_{3i})/(2\, a)$$

This is the familiar form we introduced earlier in the course and can be arranged to a double angle sine formula to solve for θ_{1i} , the actuation joint for each limb. This approach does not require a solution for θ_{2i} .

Inverse kinematics for picker robot

It is possible that the target frame may fall outside the robot's reach; thus, we must examine the special cases:

- *Generic solution* - circle of link AB intersects the BC sphere at two points, giving two solutions.
- *Singular solution* - circle tangent to sphere resulting in one solution.
- *Singular solution* - circle lies on sphere -- physically unrealistic case!
- *No solution* - circle and sphere do not intersect

Kinematics summary: parallel robots

- Inverse kinematics particularly useful, and easier to derive.
- Parallel robots use invkin to program configurations
- Commercial parallel robots are symmetrical.
- Grubler's Criterion does not readily apply to this class of complex mechanisms, because of designed redundancies and special constraints.
- Parallel robots are inherently stiffer with less inertia; thus, they can move much faster.