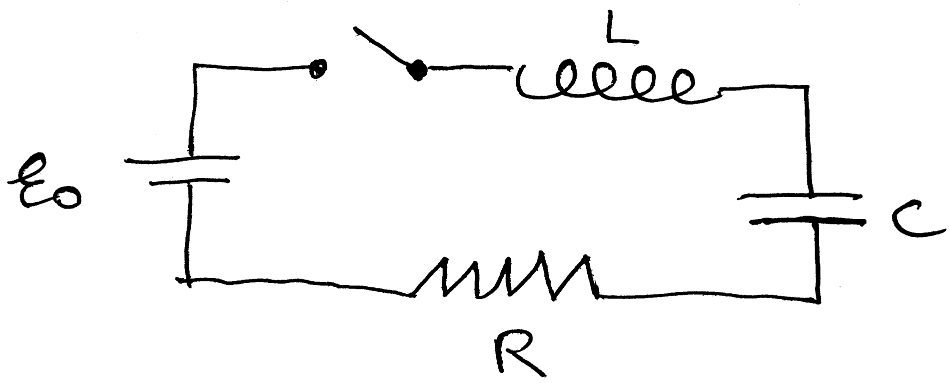
Example: RLC circuit



- When the switch is closed,

$$0 = \mathcal{E}_0 - L\frac{dI}{dt} - \frac{Q}{C} - IR$$

emf of battery

back emf due to inductor

voltage drop across capacitor

voltage drop across resistor.

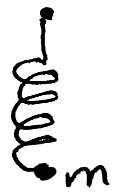
- Using $I(t) = \dfrac{dQ(t)}{dt}$

$$0 = \mathcal{E}_0 - L\frac{d^2Q(t)}{dt^2} - \frac{1}{C}Q(t) - R\frac{dQ(t)}{dt}$$

second order differential equation for $Q(t)$

$$\Rightarrow L\frac{d^2Q(t)}{dt^2} = \mathcal{E}_0 - R\frac{dQ(t)}{dt} - \frac{1}{C}Q(t)$$

• Mathematically, this takes the same form as that for a <u>damped</u> <u>simple harmonic oscillator</u> — a particle attached to a spring



If we displace the particle from equilibrium, by an amount x, $F = -kx$ (− sign means restoring force)

$$\underbrace{m \frac{d^2 x(t)}{dt^2}}_{ma} = \text{Force}$$

$$= F_0 - b\frac{dx(t)}{dt} - kx(t)$$

F_0: some constant force

$b\frac{dx(t)}{dt}$: damping term proportional to speed

$kx(t)$: spring restoring force

<u>So</u>: • $L \dfrac{d^2 Q(t)}{dt^2} \leftrightarrow m \dfrac{d^2 x(t)}{dt}$,

m is a measure of the resistance to a change of speed ($= \frac{dx(t)}{dt}$) for a particle when a force is applied, L is a measure of the resistance to change of current $\left(= \frac{dQ(t)}{dt}\right)$ when an emf is applied.

- $-b\frac{dx(t)}{dt}$ is an energy loss term for a particle (e.g frictional force)

as is $I\frac{dQ(t)}{dt} = IR$, relates to energy loss in a resistor.

- Comparison of the remaining terms

shows $-k\,x(t) \leftrightarrow \frac{1}{C}\,Q(t)$,

a capacitor is the mechanical analogue of a spring

$$E_{spring} = \frac{1}{2}kx^2$$
$$E_{capacitor} = \frac{1}{2}\frac{Q^2}{C}.$$

• In the case of a mass on a spring, once displaced, the mass oscillates up and down, and energy is continually being transferred between kinetic energy and potential energy (stored in the spring). If thee is friction (damping), the oscillations will die down.

In the electrical case, we will see that energy oscillates between being stored in the electric field of the capacitor and the magnetic field of the inductor. If thee is a resistor present, the energy is lost (as heat) and the oscillations die down.

Solving the differential equation

$$\frac{d^2Q(t)}{dt^2} = \frac{\mathcal{E}_0}{L} - \frac{R}{L}\frac{dQ(t)}{dt} - \frac{1}{LC}Q(t).$$

- First consider the case $R=0$, $\mathcal{E}_0 = 0$, but assume there is a current already established in the circuit

Then

$$L\frac{d^2Q(t)}{dt^2} = -\frac{1}{C}Q(t)$$

$$\Rightarrow \frac{d^2Q(t)}{dt^2} = -\frac{1}{LC}Q(t).$$

Solution:

$$Q(t) = Q_0 \cos(\omega t + \phi)$$

with $\omega = \frac{1}{\sqrt{LC}}$, so the charge on the capacitor oscillates between $+Q_0$ and $-Q_0$ with

[Check: $Q(t) = Q_0 \cos(\omega t + \phi)$

$$\frac{dQ(t)}{dt} = -\omega Q_0 \sin(\omega t + \phi)$$

$$\frac{d^2Q(t)}{dt^2} = -\omega^2 Q_0 \cos(\omega t + \phi)$$

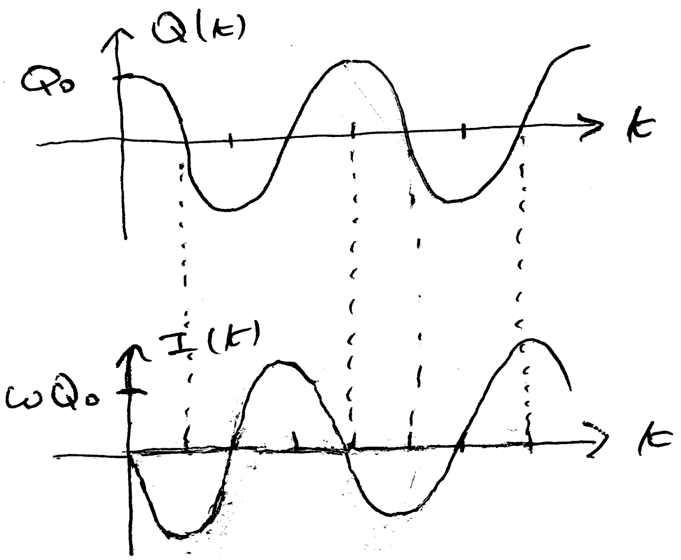
So $\dfrac{d^2Q(t)}{dt^2} = -\dfrac{1}{LC} Q(t)$

provided $\omega^2 = \dfrac{1}{LC}$].

The current is then

$$I(t) = \frac{dQ(t)}{dt} = -\omega Q_0 \sin(\omega t + \phi)$$

If $\phi = 0$, then:



- When Q is maximum (capacitor fully charged), $I = 0$.
- When I maximum, capacitor fully discharged.

- The charge on the capacitor and the current through the inductor are 90° degrees out of phase.

Energy is moving back and forth between being stored in the electric field of the capacitor (analogue of mechanical energy stored in a stretched spring) and being stored in the magnetic field of the inductor (analogue of kinetic energy for a particle attached to a spring).

Since $R=0$, there is no loss of energy.

- Here, we have assumed a real solution $Q(t) = Q_0 \cos(\omega t + \phi)$ to the differential equation

$$\frac{d^2 Q(t)}{dt^2} = -\frac{1}{LC} Q(t).$$

- However, the equation also admits complex solutions

$$Q(t) = Q_0 \, e^{i(\omega t + \phi)}$$

if $\omega = \frac{1}{\sqrt{LC}}$

[Check: $\frac{dQ(t)}{dt} = i\omega \, Q_0 \, e^{i(\omega t + \phi)}$

$$\frac{d^2 Q(t)}{dt^2} = (i\omega)^2 Q_0 \, e^{i(\omega t + \phi)}.$$

This is equal to $-\frac{1}{LC} Q(t)$ if $-\omega^2 = -\frac{1}{LC}$].

It is actually <u>much</u> <u>easier</u> to work with the complex valued solutions and take the Real part at the end of the calculation.

<u>Reason</u>: if we take

$$Q(t) = Q_0 \, e^{i(\omega t + \phi)},$$

$$\mathrm{Re}\, Q(t) = Q_0 \cos(\omega t + \phi)$$

Then when we compute derivatives, have to differentiate sines and cosines

e.g $I(t) = \frac{dQ(t)}{dt}$

$= \frac{d}{dt} Q_0 \cos(\omega t + \phi)$

$= Q_0 (-\omega) \sin(\omega t + \phi).$

But: if we differentiate the complex valued solution first and <u>then</u> take the real part, we get the same answer:

$$Q(t) = Q_0 e^{i(\omega t + \phi)}$$

$$\frac{dQ(t)}{dt} = i\omega Q_0 e^{i(\omega t + \phi)}$$

$$Re\left(\frac{dQ(t)}{dt}\right) = Re\left(i\omega Q_0 \left(\cos(\omega t+\phi) + i \sin(\omega t+\phi)\right)\right)$$

$$= Re\left(i\omega Q_0 \cos(\omega t+\phi) - \omega Q_0 \sin(\omega t+\phi)\right)$$

$$= -\omega Q_0 \sin(\omega t + \phi),$$

the same answer we got starting with a <u>real</u> solution for $Q(t)$.

- Taking the real part and

then differentiating yields the same answer as differentiating and <u>then</u> taking the real part. The <u>latter</u> is <u>much easier</u> as

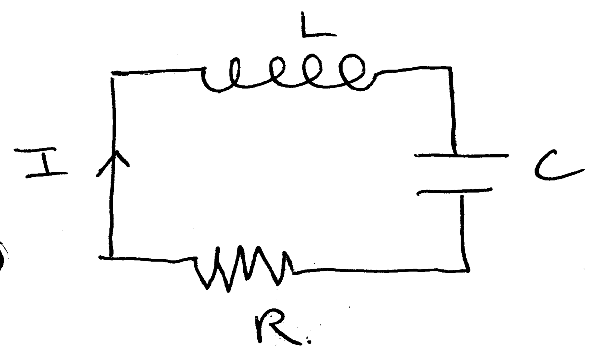
$$\frac{d}{dt} e^{i(\omega t+\phi)} = i\omega e^{i(\omega t+\phi)},$$

we don't have to worry about differentiating sines and cosines.

- WE WILL USE COMPLEX NOTATION FROM NOW ON i.e. TAKE THE REAL PART AT THE END AFTER ALL DERIVATIVES HAVE BEEN DONE.

The case $R \neq 0$

Again assume $\mathcal{E}_0 = 0$ (current already established in circuit e.g by induction.



Then
$$\frac{d^2 Q(t)}{dt^2} = -\frac{R}{L}\frac{dQ(t)}{dt} - \frac{1}{LC}Q(t) \qquad (1)$$

We expect a solution of the form
$$Q(t) = Q_0 \, e^{-\alpha t} \, e^{i(\omega t + \phi)}$$

damping term due to energy loss in resistor

oscillatory term - energy moving between capacitor and inductor.

Then
$$\frac{dQ(t)}{dt} = -\alpha Q_0 e^{-\alpha t} e^{i(\omega t + \phi)} + i\omega Q_0 e^{-\alpha t} e^{i(\omega t + \phi)}$$
$$e^{-\alpha t} e^{i(\omega t + \phi)}$$

$$\frac{d^2Q(t)}{dt^2} = (-\alpha + i\omega)^2 Q_0 e^{-\alpha t} e^{i(\omega t + \phi)}.$$

Removing a factor of $Q_0 e^{-\alpha t} e^{i(\omega t+\phi)}$ from both sides of (1)

$$(-\alpha + i\omega)^2 = -\frac{R}{L}(-\alpha + i\omega) - \frac{1}{LC}$$

$$\alpha^2 - 2i\alpha\omega - \omega^2 = \alpha\frac{R}{L} - i\omega\frac{R}{L} - \frac{1}{LC}.$$

Equating terms with no factors of i:

$$\alpha^2 - \omega^2 = \alpha\frac{R}{L} - \frac{1}{LC} \qquad \text{— (1)}$$

Equating factors with i:

$$-2i\alpha\omega = -i\omega\frac{R}{L}$$

$$\Rightarrow \boxed{\alpha = \frac{1}{2}\frac{R}{L}}$$

Substitute into (1)

$$\omega^2 = \alpha^2 - \alpha\frac{R}{L} + \frac{1}{LC}$$

$$= \frac{1}{4}\frac{R^2}{L^2} - \frac{1}{2}\frac{R}{L}\frac{R}{L} + \frac{1}{LC}$$

$$= \frac{1}{LC} - \frac{1}{4}\frac{R^2}{L^2}$$

NOTE: this only makes sense

if $\frac{1}{LC} > \frac{1}{4}\frac{R^2}{L^2}$, as ω^2 can't be negative.

So: $\boxed{\text{if } \frac{1}{LC} > \frac{1}{4}\frac{R^2}{L^2}}$

$$Q(t) = Q_0 e^{-\frac{1}{2}\frac{R}{L}t} e^{i(\omega t + \phi)}$$

with $\omega = \sqrt{\frac{1}{LC} - \frac{1}{4}\frac{R^2}{L^2}}$

(or the <u>real</u> part of this).

If $\phi = 0$,

$$\text{Re } Q(t) = Q_0 e^{-\frac{1}{2}\frac{R}{L}t} \cos \omega t$$

↑ damped oscillations.

. What if $\boxed{\frac{1}{LC} < \frac{1}{4}\frac{R^2}{L^2}}$?

Well $\sqrt{\frac{1}{LC}}$ is related to the rate of oscillation, and $\frac{1}{2}\frac{R}{L}$ determines how fast the oscillations die off

$\frac{1}{2}\frac{R}{L}t$

It could be that the damping kills off the current before it can oscillate. So let's try a purely damped solution when

$$\frac{1}{LC} < \frac{1}{4}\frac{R^2}{L^2},$$

$$Q(t) = Q_0 e^{-\alpha t}$$

$$\frac{dQ(t)}{dt} = -\alpha Q_0 e^{-\alpha t}$$

$$\frac{d^2 Q(t)}{dt^2} = (-\alpha)^2 Q_0 e^{-\alpha t}.$$

Substituting into the equation

$$\alpha^2 Q_0 e^{-\alpha t} = \left(-\frac{R}{L}\right)(-\alpha) Q_0 e^{-\alpha t} - \frac{1}{LC} Q_0 e^{-\alpha t}$$

$$\Rightarrow \quad 0 = \alpha^2 - \alpha\frac{R}{L} + \frac{1}{LC}$$

This is a quadratic equation.

(If $0 = ax^2 + bx + c$, $x = \frac{1}{2a}\left[-b \pm \sqrt{b^2 - 4ac}\right)$

So $\alpha = \frac{1}{2}\left(\frac{R}{L} \pm \sqrt{\frac{R^2}{L^2} - \frac{4}{LC}}\right)$

The square root only makes sense if $\frac{R^2}{L^2} - \frac{4}{LC} > 0$

$$\Rightarrow \frac{1}{4}\frac{R^2}{L^2} > \frac{1}{LC},$$

which is what applied.

So for $\frac{1}{LC} < \frac{1}{4}\frac{R^2}{L^2}$, we get a fully damped solution:

$$Q(t) = Q_0 e^{-\alpha t}$$

with $\alpha = \frac{1}{2}\frac{R}{L} \pm \sqrt{\frac{1}{4}\frac{R^2}{L^2} - \frac{1}{LC}}$.

The current damps out before oscillations can occur.

This is called <u>OVER-DAMPING</u>

Energy conservation in EM

- Recall: charge conservation could be expressed via the "continuity equation"

$$\frac{\partial \rho(\vec{r},t)}{\partial t} + \vec{\nabla}\cdot\vec{j}(\vec{r},t) = 0$$

- where $\rho(\vec{r},t)$ = charge density
  = charge per unit volume

and $\vec{j}(\vec{r},t)$ = current density
  = current per unit cross-sectional area.

-

Reason: consider a volume V with surface S.

The charge inside S at time t is

$$\int_V \rho(\vec{r},t)\, \underbrace{d^3\vec{r}}_{dx\,dy\,dz\,=\,dV}$$

• There is a similar equation expressing <u>energy conservation</u>: if energy is flowing out through a closed surface S, the the energy inside the surface S must decrease. In EM, we have seen there is energy associated with electric and magnetic fields. So let's see what the equation governing conservation of this energy looks like.

• We have seen that for $\vec{E}$ and $\vec{B}$ fields, the <u>energy densities</u> (energy per unit volume) are

$$u_{elec}(\vec{r},t) = \frac{\epsilon_0}{2}\vec{E}(\vec{r},t)\cdot\vec{E}(\vec{r},t)$$

$$u_{mag}(\vec{r},t) = \frac{\epsilon_0 c^2}{2}\vec{B}(\vec{r},t)\cdot\vec{B}(\vec{r},t)$$

Then

$$\frac{d}{dt}\int_V \rho(\vec{r},t)\, d^3\vec{r}$$

$$= \int_V \frac{\partial \rho(\vec{r},t)}{\partial t}\, d^3\vec{r}$$

$$= -\int_V \vec{\nabla}\cdot\vec{J}(\vec{r},t)\, d^3\vec{r}$$

$$= -\underbrace{\oint_S \vec{J}(\vec{r},t)\cdot d\vec{S}}_{\text{current through } S} \quad \text{by Gauss's theorem}$$

ie rate of change of charge inside S = current through S.

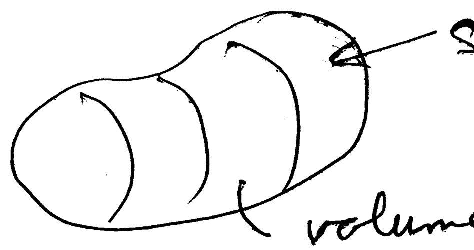
The minus sign: if the charge inside S is decreasing, $\frac{d}{dt}\int_V \rho(\vec{r},t)\, d^3\vec{r}$ is negative, whilst the current out through S $\oint \vec{J}(\vec{r},t)\cdot d\vec{S}$ is positive.

• So, the energy inside a closed surface S (enclosing a volume V) at time t

$$U(t) = \int_V \left(u_{elec}(\vec{r},t) + u_{mag}(\vec{r},t) \right) \underbrace{d^3\vec{r}}_{dV}$$

$$= \frac{\varepsilon_0}{2} \int_V \vec{E}(\vec{r},t) \cdot \vec{E}(\vec{r},t)\, d^3\vec{r}$$

$$+ \frac{\varepsilon_0 c^2}{2} \int_V \vec{B}(\vec{r},t) \cdot \vec{B}(\vec{r},t)\, d^3\vec{r}$$



• Using $\frac{\partial}{\partial t}(\vec{E}\cdot\vec{E}) = 2\,\vec{E}\cdot\frac{\partial \vec{E}}{\partial t}$

and $\frac{\partial}{\partial t}(\vec{B}\cdot\vec{B}) = 2\,\vec{B}\cdot\frac{\partial \vec{B}}{\partial t}$,

the rate of change of energy inside surface S is

$$\frac{dU(t)}{dt} = \varepsilon_0 \int_V \vec{E}(\vec{r},t) \cdot \frac{\partial \vec{E}(\vec{r},t)}{\partial t}\, d^3\vec{r}$$

$$+ \varepsilon_0 c^2 \int \vec{B}(\vec{r},t) \cdot \frac{\partial \vec{B}(\vec{r},t)}{\partial t}\, d^3\vec{r}.$$

(*)

Now we can use Maxwell's equations:

Ampere's law:

$$\vec{\nabla} \times \vec{B}(\vec{r},t) = \frac{\vec{J}(\vec{r},t)}{\varepsilon_0 c^2} + \frac{1}{c^2}\frac{\partial \vec{E}(\vec{r},t)}{\partial t}$$

$$\Rightarrow \frac{\partial \vec{E}(\vec{r},t)}{\partial t} = c^2 \vec{\nabla} \times \vec{B}(\vec{r},t) - \frac{1}{\varepsilon_0}\vec{J}(\vec{r},t).$$

Faraday's law:

$$\vec{\nabla} \times \vec{E}(\vec{r},t) = -\frac{\partial \vec{B}(\vec{r},t)}{\partial t}$$

$$\Rightarrow \frac{\partial \vec{B}(\vec{r},t)}{\partial t} = -\vec{\nabla} \times \vec{E}(\vec{r},t).$$

Substituting into (*):

$$\frac{dU(t)}{dt} = \varepsilon_0 c^2 \int_V \vec{E}(\vec{r},t) \cdot \vec{\nabla} \times \vec{B}(\vec{r},t)\, d^3\vec{r} - \int_V \vec{E}(\vec{r},t) \cdot \vec{J}(\vec{r},t)\, d^3\vec{r} - \varepsilon_0 c^2 \int_V \vec{B}(\vec{r},t) \cdot \vec{\nabla} \times \vec{E}(\vec{r},t)\, d^3\vec{r}$$

Using the vector identity

$$\vec{\nabla} \cdot (\vec{E} \times \vec{B}) = \vec{B} \cdot (\vec{\nabla} \times \vec{E}) - \vec{E} \cdot (\vec{\nabla} \times \vec{B}),$$

the first and last terms combine to give

$$\frac{dU(t)}{dt} = -\varepsilon_0 c^2 \int_V \vec{\nabla} \cdot \left(\vec{E}(\vec{r},t) \times B(\vec{r},t) \right) d^3\vec{r} - \int_V \vec{E}(\vec{r},t) \cdot \vec{J}(\vec{r},t)\, d^3\vec{r}$$

$$= -\varepsilon_0 c^2 \oint \left(\vec{E}(\vec{r},t) \times \vec{B}(\vec{r},t) \right) \cdot d\vec{S} - \int_V \vec{E}(\vec{r},t) \cdot \vec{J}(\vec{r},t)\, d^3\vec{r}.$$

Recall: this is the rate of change of energy inside the closed surface

- Interpretation of first term on RHS:

$$\varepsilon_0 c^2 \oint_S \left(\vec{E}(\vec{r},t) \times \vec{B}(\vec{r},t) \right) \cdot d\vec{S}$$

is the energy per unit time flowing through the surface S.

- Interpretation of the second term on the RHS:

$$\int_V \vec{E}(\vec{r},t) \cdot \vec{J}(\vec{r},t)\, d^3\vec{r}$$

is work done per unit time by the $\vec{E}$ field on any charges inside S that are moving.

<u>Reason</u>: if a charge q is moving, its displacement in time dt is $d\vec{r} = \vec{v}\,dt$.

If it is moving in $\vec{E}$ and $\vec{B}$ fields, the force on it is the Lorentz force

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

$\Rightarrow$ the work done by the $\vec{E}$ and $\vec{B}$ fields in time dt is

$$dW = \vec{F} \cdot d\vec{r}$$
$$= (q\vec{E} + q\,\vec{v} \times \vec{B}) \cdot \vec{v}\, dt$$
$$= q\,\vec{E} \cdot \vec{v}\, dt$$

since $(\vec{v} \times \vec{B}) \cdot \vec{v} = 0$
(magnetic field does no work on a charged particle

$$\Rightarrow \quad \frac{dW}{dt} = q\,\vec{E} \cdot \vec{v}$$

For a continuous charge distribution

$$q \to \int \rho\, d^3\vec{r}, \quad \rho\vec{v} \to \vec{J}$$

$$\Rightarrow \quad \frac{dW}{dt} = \int_V \vec{E} \cdot \vec{J}\, d^3\vec{r}$$

- So, going back to our equation

$$\underbrace{\frac{d}{dt}\int_V u(\vec{r},t)\, d^3\vec{r}}_{\text{energy inside S in } \vec{E} \text{ and } \vec{B} \text{ fields}, \; u = u_{elec} + u_{mag}}$$

$$= \underbrace{-\varepsilon_0 c^2 \oint_S \left(\vec{E}(\vec{r},t) \times \vec{B}(\vec{r},t)\right) \cdot d\vec{S}}_{\text{energy flow through S}}$$

$$\underbrace{- \int_V \vec{E}(\vec{r},t) \cdot \vec{J}(\vec{r},t)\, d^3\vec{r}}_{\text{work done by } \vec{E} \text{ field on moving charges inside S}}$$

Signs:

If $\oint_S (E \times \vec{B}) \cdot d\vec{S}$ and $\int_V \vec{E} \cdot \vec{J} \cdot d^3\vec{r}$ are positive, the energy inside S decreases

$\Rightarrow \frac{d}{dt}\int_V u(\vec{r},t)\, d^3\vec{r}$ is negative

- The vector

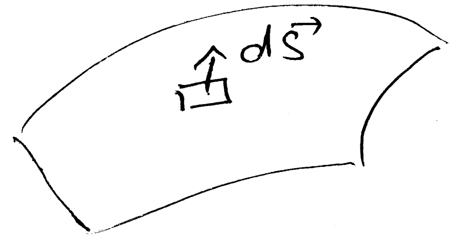
$$\varepsilon_0 c^2 \, \vec{E}(\vec{r},t) \times \vec{B}(\vec{r},t)$$

is the energy flow per unit time per unit cross sectional area as a result of the presence of $\vec{E}$ and $\vec{B}$ fields.

- It is called the Poynting vector, denoted $\vec{S}(\vec{r},t)$:

$$\boxed{\vec{S}(\vec{r},t) = \varepsilon_0 c^2 \, \vec{E}(\vec{r},t) \times \vec{B}(\vec{r},t)}$$

NOTE, do not confuse with $d\vec{S}$,
- an element of surface



NOTE: in a charge capacitor, $\vec{B} = 0$
$\Rightarrow$ Poynting vector $= 0$, no flow of energy, But $u_{elec} = \frac{\varepsilon_0}{2} \vec{E} \cdot \vec{E} \neq 0$

• So our energy conservation equation is

$$\frac{d}{dt}\int_V u(\vec{r},t)\, d^3\vec{r}$$

↑ energy density

$$= -\oint_S \vec{S}(\vec{r},t)\, d^3\vec{r}$$

↑ Poynting vector

$$-\int_V \vec{E}(\vec{r},t)\cdot\vec{J}(\vec{r},t)\, d^3\vec{r}$$

In a region of space where there are no currents, $\vec{J}=0$

$$\Rightarrow \int_V \frac{\partial u(\vec{r},t)}{\partial t}\, d^3\vec{r}$$

$$= -\oint_S \vec{S}(\vec{r},t)\cdot d\vec{S}$$

↑ surface element

$$= \int_V \vec{\nabla}\cdot\vec{S}(\vec{r},t)$$

via Gauss's theorem

$$\Rightarrow \frac{\partial u(\vec{r},t)}{\partial t} + \vec{\nabla}\cdot\vec{S}(\vec{r},t) = 0$$

↑ energy conservation

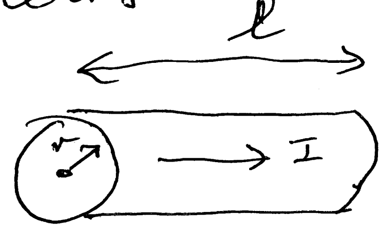
Compare with <u>charge</u> conservation

$$\frac{\partial \rho(\vec{r},t)}{\partial t} + \vec{\nabla}\cdot\vec{J}(\vec{r},t) = 0$$

• Example of use of the Poynting vector — power dissipated in a resistor.

• The voltage drop across a resistor is $\Delta V = I R$
R ← resistance

• Consider a resistor with the following geometry

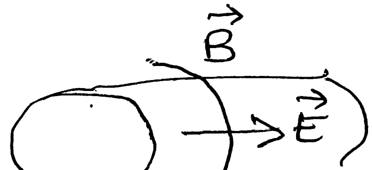


There is an electric field in the resistor driving the current,

$$\Delta V = E \cdot \ell$$

where $E = |\vec{E}|$ is the magnitude of the electric field.

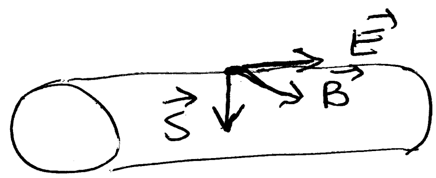
The current also produces a magnetic field which circulates around the wire



So at any point on the surface of the resistor, the Poynting vector

$$\vec{S} = \varepsilon_0 c^2 \vec{E} \times \vec{B}$$ is directed

inwards $\Rightarrow$ there is a flow of energy into the resistor.



This energy is dissipated in the form of heat.

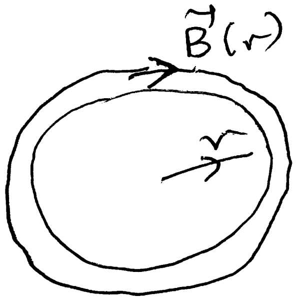
[Note: if $R = 0$, there is no voltage drop across the resistor $\Rightarrow \vec{E} = 0$, and so $\vec{S} = 0$, no power is dissipated].

Since $\vec{E}$ and $\vec{B}$ are perpendicular

$$|\vec{S}| = \varepsilon_0 c^2 |\vec{E}| |\vec{B}|.$$

We have already seen $|\vec{E}| = \frac{\Delta V}{\ell}$.

To work out $\vec{B}$ at the surface of the wire, we use Ampere's law:



← current into page

- circulation of $\vec{B}$ around surface of wire

$$= \frac{\text{current enclosed}}{\epsilon_0 c^2}$$

ie $B(r) \cdot 2\pi r = \dfrac{I}{\epsilon_0 c^2}$

$$\Rightarrow \quad B(r) = \frac{I}{2\pi \epsilon_0 c^2 r}$$

- So $|\vec{S}| = \epsilon_0 c^2 \dfrac{\Delta V}{\ell} \dfrac{I}{2\pi \epsilon_0 c^2 r}$

$$= \frac{\Delta V \cdot I}{2\pi r \ell}.$$

<u>Now</u>: $|\vec{S}|$ is the amount of energy per unit time (i.e. <u>power</u>) per unit surface area of

So the determine the total power dissipated by the resistor, we have to multiply by the surface area of the resistor, which is $2\pi r l$.

So power dissipated by the resistor is

$$P = |\vec{S}| \, 2\pi r l$$

$$= \frac{I \Delta V}{2\pi r l} \, 2\pi r l$$

$$= I \Delta V.$$

Using $\Delta V = I R$, this is also equal to

$$P = I^2 R$$, the usual result

We can also express it as

$$P = \frac{(\Delta V)^2}{R}$$